



# **INTERNATIONAL JOURNAL OF NEUTROSOPHIC SCIENCE**

---

**Volume 10, 2020**

**Editor(s) in Chief: Broumi Said & Florentin Smarandache**

**ISSN(ONLINE): 2690-6805  
ISSN(PRINT) : 22692-6148**



## Table of Content

### International Journal of Neutrosophic Science (IJNS)

Items	Page Number
Table of Contents	2
Editorial Board	4
Aim and Scope	6
Topics of Interest	6
<b>ISSUE 1</b>	
<i>Introduction to Neutrosophic Hypernear-rings</i> <i>M.A. Ibrahim, A.A.A. Agboola</i>	9-22
A Note on Neutrosophic Polynomials and Some of Its Properties <i>Somen Debnath and Anjan Mukherjee</i>	23-35
The Polar form of a Neutrosophic Complex Number <i>Riad K. Al-Hamido, Mayas Ismail, Florentin Smarandache</i>	36-44
Nonagonal Neutrosophic Linear Non-Linear Numbers, Alpha Cuts and Their Applications using TOPSIS <i>Muhammad Naveed Jafar, Ezgi TÜRKARSLAN, Ali Hamza, Sara Farooq</i>	45-64
An Expanded Model of Unmatter from Neutrosophic Logic perspective: Towards Matter-Spirit Unity View <i>Victor Christianto, Robert N. Boyd, and Florentin Smarandache</i>	65-72
<b>ISSUE 2</b>	
A Note on Single Valued Neutrosophic Sets in Ordered Groupoids <i>M. Al-Tahan, B. Davvaz, M. Parimala</i>	73-83
On Finite NeutroGroups of Type-NG[1,2,4] <i>Agboola A.A.A</i>	84-95
A Review of Fuzzy Soft Topological Spaces, Intuitionistic Fuzzy Soft Topological Spaces and Neutrosophic Soft Topological Spaces <i>M. Parimala, M.Karthika and F. Smarandache</i>	96-104
Interval-Valued Triangular Neutrosophic Linear Programming Problem <i>Bhimraj Basumatary and Said Broumi</i>	105-115
TOPSIS BY USING PLITHOGENIC SET IN COVID-19 DECISION MAKING <i>C. Sankar, R. Sujatha, and D. Nagarajan</i>	116-125

**International Journal of Neutrosophic Science (IJNS) ABSTRACTED/INDEXED IN**

**Google Scholar**



**Index Copernicus ( ICI World of Journals)**



**BASE Search**



**Microsoft Academic**



**Advanced Science Index (ASI)**



**EuroPub**



**WorldCat**





## **Editorial Board**

### **Editor in Chief**

**Dr. Broumi Said** Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco

**Prof. Dr. Florentin Smarandache** Departement of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM, 87301, United States

### **Editorial Board Members :**

Francisco Gallego Lupianez, Department of Mathematics, Universidad Complutense, Madrid, Spain. ([fg\\_lupianez@mat.ucm.es](mailto:fg_lupianez@mat.ucm.es))

Cengiz Kahraman, Istanbul Technical University, Department of Industrial Engineering 34367 Macka/Istanbul/Turkey ([kahramanc@itu.edu.tr](mailto:kahramanc@itu.edu.tr)).

Selçuk Topal, Department of Mathematics, Bitlis Eren University, Turkey ([s.topal@beu.edu.tr](mailto:s.topal@beu.edu.tr)).

Muhammad Aslam, Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia ([aslam\\_ravian@hotmail.com](mailto:aslam_ravian@hotmail.com); [magmuhammad@kau.edu.sa](mailto:magmuhammad@kau.edu.sa)).

Philippe Schweizer, Independent researcher, Av. de Lonay 11, 1110 Morges, Switzerland ([flippe2@gmail.com](mailto:flippe2@gmail.com)).

Amira Salah Ahmed Ashour, Tanta University, Tanta, Egypt ([amirasashour@yahoo.com](mailto:amirasashour@yahoo.com); [amira.salah@f-eng.tanta.edu.eg](mailto:amira.salah@f-eng.tanta.edu.eg)).

Peide Liu, School of Management Science and Engineering, Shandong University of Finance and Economics, China ([peide.liu@gmail.com](mailto:peide.liu@gmail.com)).

Jun Ye, Institute of Rock Mechanics, Ningbo University, Ningbo, P. R, China ([yehjun@aliyun.com](mailto:yehjun@aliyun.com); [yeyun1@nbu.edu.cn](mailto:yeyun1@nbu.edu.cn)).

Yanhui Guo, Department of Computer Science, University of Illinois at Springfield, USA ([yguo56@uis.edu](mailto:yguo56@uis.edu) [guoyanhui@gmail.com](mailto:guoyanhui@gmail.com)).

İrfan DELİ, Kilis 7 Aralık University, Turkey ([irfandeli20@gmail.com](mailto:irfandeli20@gmail.com)).

Vakkas Uluçay, Kilis 7 Aralık University, Turkey ([vulucay27@gmail.com](mailto:vulucay27@gmail.com)).

Chao Zhang, Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, School of Computer and Information Technology, Shanxi University, China ([czhang@sxu.edu.cn](mailto:czhang@sxu.edu.cn)).

Xindong Peng, Shaoguan University, China ([952518336@qq.com](mailto:952518336@qq.com)).

Surapati Pramanik, Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, Dist-North 24 Parganas, West Bengal, PIN-743126, India ([surapati.math@gmail.com](mailto:surapati.math@gmail.com)).

M. Lathamaheswari, Department of Mathematics Hindustan Institute of Technology and Science, Chennai-603203, India ([lathamax@gmail.com](mailto:lathamax@gmail.com); [mlatham@hindustanuniv.ac.in](mailto:mlatham@hindustanuniv.ac.in)).

Lemnaouar Zedam, Department of Mathematics, University of M'sila, Algeria ([lemnaouar.zedam@univ-msila.dz](mailto:lemnaouar.zedam@univ-msila.dz)).

S. A. Edalatpanah, Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran ([saedalatpanah@gmail.com](mailto:saedalatpanah@gmail.com)).

Liu Chun Feng, Shenyang Aerospace University, China ([liuchunfang1112@163.com](mailto:liuchunfang1112@163.com)).

Wadei faris Mohammed Al-omeri, Department of Mathematics, Faculty of Science, Al-Balqa Applied University, Salt 19117, Jordan ([wadeimoon1@hotmail.com](mailto:wadeimoon1@hotmail.com), [wadeialomeri@bau.edu.jo](mailto:wadeialomeri@bau.edu.jo)).

Abhijit Saha, Department of Mathematics, Techno College of Engineering Agartala Maheshkhola-799004, Tripura, India ([abhijit84.math@gmail.com](mailto:abhijit84.math@gmail.com)).

D.Nagarajan, Department of Mathematics, Hindustan Institute of Technology & Science, India ([dnagarajan@hindustanuniv.ac.in](mailto:dnagarajan@hindustanuniv.ac.in)).

Prem Kumar Singh, Amity Institute of Information Technology, Amity University-Sector 125 Noida-201313, Uttar Pradesh-India ([premsingh.csjm@gmail.com](mailto:premsingh.csjm@gmail.com)).

Avishek Chakraborty, Department of Basic Science, University/College- Narula Institute of Technology Under MAKAUT, India ([tirtha.avishek93@gmail.com](mailto:tirtha.avishek93@gmail.com)).

Arindam Dey, Department of Computer Science and Engineering, Saroj Mohan Institute of Technology, Hooghly 712512, West Bengal, India ([arindam84nit@gmail.com](mailto:arindam84nit@gmail.com)).

Muhammad Gulistan, Department of Mathematics & Statistics, Hazara University Mansehra, Khyber Pakhtunkhwa, Pakistan ([gulistanmath@hu.edu.pk](mailto:gulistanmath@hu.edu.pk)).

Mohsin Khalid, The University of Lahore, Pakistan ([mk4605107@gmail.com](mailto:mk4605107@gmail.com)).

Le Hoang Son, PhD, Vietnam National University, Hanoi, Vietnam, ([sonlh@vnu.edu.vn](mailto:sonlh@vnu.edu.vn)).

Kishore Kumar P.K, Department of Information Technology, Al Musanna College of Technology, Sultanate of Oman ([kishore2982@gmail.com](mailto:kishore2982@gmail.com)).

Mohamed Talea, Laboratory of Information processing, Faculty of Science Ben M'Sik, Morocco ([taleamohamed@yahoo.fr](mailto:taleamohamed@yahoo.fr)).

Assia Bakali, Ecole Royale Navale, Casablanca, Morocco ([assiabakali@yahoo.fr](mailto:assiabakali@yahoo.fr)).

Tahir Mahmood, Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan ([tahirbakhath@iiu.edu.pk](mailto:tahirbakhath@iiu.edu.pk)).

Faruk KARAASLAN, Çankırı Karatekin University, Faculty of Sciences, Department of Mathematics, 18100, Çankırı, TURKEY ([fkaraaslan@karatekin.edu](mailto:fkaraaslan@karatekin.edu)).

Mohamed Abdel-Basset, Department of Computer Science, Zagazig University, Egypt ([analyst\\_mohamed@yahoo.com](mailto:analyst_mohamed@yahoo.com), [analyst\\_mohamed@zu.edu.eg](mailto:analyst_mohamed@zu.edu.eg)).

Riad Khider AlHamido, Department of mathematics Faculty of Sciences - Al- Furat University, Syria ([riad-hamido1983@hotmail.com](mailto:riad-hamido1983@hotmail.com)).

Fahad Mohammed Alsharari, Mathematics Department, College of Science and Human Studies at Hotat Sudair, Majmaah University, Saudi Arabia ([f.alsharari@mu.edu.sa](mailto:f.alsharari@mu.edu.sa)).

Maikel Leyva Vázquez, Universidad Politécnica Salesiana, Ecuador ([mleyvaz@gmail.com](mailto:mleyvaz@gmail.com)).

Ir. Victor Christianto, DDiv. (associate of NSIA), Satyabhakti Advanced School of Theology - Jakarta Chapter, Indonesia ([victorchristianto@gmail.com](mailto:victorchristianto@gmail.com)).

Xiaohong Zhang, Shaanxi University of Science and Technology, China ([zhangxiaohong@sust.edu.cn](mailto:zhangxiaohong@sust.edu.cn), [zxhonghz@263.net](mailto:zxhonghz@263.net)).

Huda E. Khalid, Head of the Scientific Affairs and Cultural Relations, Presidency of Telafer University ,Iraq ([hodaesmail@yahoo.com](mailto:hodaesmail@yahoo.com), [dr.huda-ismael@uotelafer.edu.iq](mailto:dr.huda-ismael@uotelafer.edu.iq)).

Ranjan Kumar, Department of Mathematics, National Institute of Technology, Adityapur, Jamshedpur, 831014, India. ([ranjank.nit52@gmail.com](mailto:ranjank.nit52@gmail.com)).

Choonkil Park, Department of Mathematics, Hanyang University, Republic of Korea ([baak@hanyang.ac.kr](mailto:baak@hanyang.ac.kr)).

Shahzaib Ashraf, Department of Mathematic Abdul Wali Khan University, Mardan 23200, Pakistan ([shahzaibashraf@awkum.edu.pk](mailto:shahzaibashraf@awkum.edu.pk)).

Madeleine Al-Tahan, Lebanese International University, Lebanon ([madeline.tahan@liu.edu.lb](mailto:madeline.tahan@liu.edu.lb)).

Rafif alhabib, Department of mathematical statistic, Faculty of Sciences, albaath University, Syria. ([rafif.alhabib85@gmail.com](mailto:rafif.alhabib85@gmail.com))

Giorgio Nordo, MIFT - Department of Mathematical and Computer Science, Physical Sciences and Earth Sciences, Messina University, Italy. ([giorgio.nordo@unime.it](mailto:giorgio.nordo@unime.it))

Angelo de Oliveira, Departamento Academico de Ciencia da Computacao - Universidade Federal de Rondonia, Brasil, ([angelo@unir.br](mailto:angelo@unir.br))

A.A.A. Agboola, Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria, ([agboolaaaa@funaab.edu.ng](mailto:agboolaaaa@funaab.edu.ng))

A. A. Salama, Department of Mathematics and Computer Science, Faculty of Science, Port Said University, Port Said, Egypt, ([drsalama44@gmail.com](mailto:drsalama44@gmail.com))

Akbar Rezaei, Department of Mathematics, Payame Noor University, P.O.Box. 19395-3697, Tehran, IRAN, ([rezaei@pnu.ac.ir](mailto:rezaei@pnu.ac.ir))

Ahmed Hatip, Department of Mathematics, Gaziantep University, Turkey, ([kollnaar5@gmail.com](mailto:kollnaar5@gmail.com))

Metawee Songsaeng, Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand, ([metawee.faith@gmail.com](mailto:metawee.faith@gmail.com))

Kavikumar Jacob, Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology Universiti Tun Hussein Onn Malaysia, 86400 Malaysia. ([kavi@uthm.edu.my](mailto:kavi@uthm.edu.my))

Kifayat Ullah, Department of Mathematics and Statistics, International Islamic University Islamabad, Pakistan

M.B.Zeina, University of Aleppo, Aleppo-Syria ([bisherzeina@alepuniv.edu.sy](mailto:bisherzeina@alepuniv.edu.sy)).

Fatimah Mahmood Mohammed, Department of Mathematic, College of Education for Pure Sciences, Tikrit University, Tikrit, IRAQ ([dr.fatimahmahmood@tu.edu.iq](mailto:dr.fatimahmahmood@tu.edu.iq))

Sadi Bayramov, Department of Algebra and Geometry, Baku State University, Baku, Azerbaijan ([baysadi@gmail.com](mailto:baysadi@gmail.com))

Qin Xin, Faculty of Science and Technology ,University of the Faroe Islands, Vestarabryggja 15, FO 100 Torshavn, Faroe Islands ([QinX@setur.fo](mailto:QinX@setur.fo))

Darjan Karabašević, Faculty of Applied Management, Economics and Finance in Belgrade - MEF, University Business Academy in Novi Sad, Serbia

Dragisa Stanujkic, Technical Faculty in Bor, University of Belgrade, Bor, Serbia, ([dstanujkic@tfbor.bg.ac.rs](mailto:dstanujkic@tfbor.bg.ac.rs))



## Aim and Scope

*International Journal of Neutrosophic Science (IJNS)* is a peer-review journal publishing high quality experimental and theoretical research in all areas of Neutrosophic and its Applications. IJNS is published quarterly. IJNS is devoted to the publication of peer-reviewed original research papers lying in the domain of neutrosophic sets and systems. Papers submitted for possible publication may concern with foundations, neutrosophic logic and mathematical structures in the neutrosophic setting. Besides providing emphasis on topics like artificial intelligence, pattern recognition, image processing, robotics, decision making, data analysis, data mining, applications of neutrosophic mathematical theories contributing to economics, finance, management, industries, electronics, and communications are promoted. Variants of neutrosophic sets including refined neutrosophic set (RNS). Articles evolving algorithms making computational work handy are welcome.

## Topics of Interest

IJNS promotes research and reflects the most recent advances of neutrosophic Sciences in diverse disciplines, with emphasis on the following aspects, but certainly not limited to:

- |  |  |
|--|--|
| <input type="checkbox"/> Neutrosophic sets   | <input type="checkbox"/> Neutrosophic algebra                      |
| <input type="checkbox"/> Neutrosophic topolog  | <input type="checkbox"/> Neutrosophic graphs                       |
| <input type="checkbox"/> Neutrosophic probabilities  | <input type="checkbox"/> Neutrosophic tools for decision making    |
| <input type="checkbox"/> Neutrosophic theory for machine learning  | <input type="checkbox"/> Neutrosophic statistics                   |
| <input type="checkbox"/> Neutrosophic numerical measures   | <input type="checkbox"/> Classical neutrosophic numbers            |
| <input type="checkbox"/> A neutrosophic hypothesis   | <input type="checkbox"/> The neutrosophic level of significance    |
| <input type="checkbox"/> The neutrosophic confidence interval  | <input type="checkbox"/> The neutrosophic central limit theorem    |
| <input type="checkbox"/> Neutrosophic theory in bioinformatics   |  |
| <input type="checkbox"/> and medical analytics   | <input type="checkbox"/> Neutrosophic tools for big data analytics |
| <input type="checkbox"/> Neutrosophic tools for deep learning  | <input type="checkbox"/> Neutrosophic tools for data visualization |
| <input type="checkbox"/> Quadripartitioned single-valued   |  |
| <input type="checkbox"/> neutrosophic sets   | <input type="checkbox"/> Refined single-valued neutrosophic sets   |
| <input type="checkbox"/> Applications of neutrosophic logic in image processing                              |  |
| <input type="checkbox"/> Neutrosophic logic for feature learning, classification, regression, and clustering |  |

- ☐ Neutrosophic knowledge retrieval of medical images
- ☐ Neutrosophic set theory for large-scale image and multimedia processing
- ☐ Neutrosophic set theory for brain-machine interfaces and medical signal analysis
- ☐ Applications of neutrosophic theory in large-scale healthcare data
- ☐ Neutrosophic set-based multimodal sensor data
- ☐ Neutrosophic set-based array processing and analysis
- ☐ Wireless sensor networks Neutrosophic set-based Crowd-sourcing
- ☐ Neutrosophic set-based heterogeneous data mining
- ☐ Neutrosophic in Virtual Reality
- ☐ Neutrosophic and Plithogenic theories in Humanities and Social Sciences
- ☐ Neutrosophic and Plithogenic theories in decision making
- ☐ Neutrosophic in Astronomy and Space Sciences

***Published and typeset in American Scientific Publishing Group (ASPG)*** is a USA academic publisher, established as LLC company on 2019 at New Orleans, Louisiana, USA. ASPG publishes online scholarly journals that are free of submission charges.

***Copyright © 2020 American Scientific Publishing Group (ASPG)***

American Scientific Publishing Group (ASPG) LLC,  
New Orleans, USA

Mailing Address: 625 Wright Ave, Gretna, LA 70056, USA

Phone: +1(504) 336-3385

Fax: +1-888-417-7465

e-mail: [manager@americaspg.com](mailto:manager@americaspg.com)

[www.americaspg.com](http://www.americaspg.com)



## Introduction to Neutrosophic Hypernear-rings

<sup>1</sup> M.A. Ibrahim and <sup>2</sup> A.A.A. Agboola

<sup>1,2</sup>Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria

muritalaibrahim40@gmail.com<sup>1</sup> and agboolaaaa@funaab.edu.ng<sup>2</sup>

### Abstract

This paper is concerned with the introduction of neutrosophic hypernear-rings. The concept of neutrosophic  $A$ -hypergroup of a hypernear-ring  $A$ , neutrosophic  $A(I)$ -hypergroup of a neutrosophic hypernear-ring  $A(I)$  and their respective neutrosophic substructures are defined. We investigate and present some interesting results arising from the study of hypernear-rings in neutrosophic environment. It is shown that a constant neutrosophic hypernear-ring in general is not a constant hypernear-ring. In addition, we consider the neutrosophic ideals, neutrosophic homomorphism and neutrosophic quotient hypernear-rings of neutrosophic hypernear-rings.

**Keywords:** neutrosophy, neutrosophic hypernear-ring, neutrosophic  $A$ -hypergroup, neutrosophic  $A(I)$ -hypergroup, neutrosophic  $A$ -subhypergroup, neutrosophic  $A(I)$ -subhypergroup, neutrosophic hyperideals, neutrosophic hypernear-ring homomorphism, neutrosophic  $A$ -hypergroup homomorphism.

## 1 Introduction

Algebraic Hyperstructures are a natural extension/generalization of classical algebraic structures. This theory was introduced in 1934 by Marty. Since then, the theory and its applications to various aspect of sciences have been extensively studied by Corsini [8,9,10], Mittas [19,20], Stratigopoulos [23] and many other authors. For instance, Dasic in [11] studied the notion of hypernear-ring. He defined hypernear-rings, as the natural generalization of near-rings, endowed with quasicanonical hypergroups  $(R, +)$  with multiplication being distributive with respect to the hyperaddition on the left side, and such that  $(R, \cdot)$  is a semigroup with bilaterally absorbing element. In [13], Gontineac called the hypernear-ring presented by Dasic as a zero symmetric hypernear-ring and he studied the concept of hypernear-ring in a more general case. Kyung *et al.* presented in [18] the notion of hyper  $R$ -sugroups of a hypernear-ring and they investigated some properties of hypernear-rings with respect to the hyper  $R$ -subgroups. For more comprehensive details on hyperstructures, the reader should see [12,18,21].

A well established branch of neutrosophic theory is the theory of neutrosophic algebraic structures. This aspect of neutrosophic theory was introduced in [24] by Kandasamy and Smarandache. They combined the elements of a given algebraic structure  $(X, \star)$  with the indeterminate element  $I$ , and, the new structure  $(X(I), \star)$  generated by  $X$  and  $I$  is called a neutrosophic algebraic structure. For more details about neutrosophic algebraic structures (see [6,14,22,25,26]). Recently, Agboola and Davvaz in [0,23,4] introduced the connections between neutrosophic set and the theory of algebraic hyperstructure. They studied neutrosophic BCI/BCK-Algebras, neutrosophic hypergroups, neutrosophic canonical hypergroups, and neutrosophic hyperrings. In this paper, we will investigate and present some interesting results arising from the study of hypernear-rings in a neutrosophic environment. This paper will add to the growing list of papers connecting algebraic hyperstructures and neutrosophic sets. More of such connections can be found in many recent publications some of which are [5,7,15,16,17].

## 2 Preliminaries

In this section, we will present some definitions and results that will be used later in the paper.

**Definition 2.1.** [18] A hypernear-ring is an algebraic structure  $(R, +, \cdot)$  which satisfies the following axioms:

1.  $(R, +)$  is a quasi canonical hypergroup (not necessarily commutative), i.e., in  $(R, +)$  the following hold:

- (a)  $x + (y + z) = (x + y) + z$  for all  $x, y, z \in R$ ;
- (b) There is  $0 \in R$  such that  $x + 0 = 0 + x = x$  for all  $x \in R$ ;
- (c) For every  $x \in R$  there exists one and only one  $x' \in R$  such that  $0 \in x + x'$ , (we shall write  $-x$  for  $x'$  and we call it the opposite of  $x$ );
- (d)  $z \in x + y$  implies  $y \in -x + z$  and  $x \in z - y$ .

If  $x \in R$  and  $A, B$  are subsets of  $R$ , then by  $A + B$ ,  $A + x$  and  $x + B$  we mean

$$A + B = \bigcup_{a \in A, b \in B} a + b, A + x = A + \{x\}, x + B = \{x\} + B.$$

- 2. With respect to the multiplication,  $(R, \cdot)$  is a semigroup having absorbing element 0, i.e.,  $x \cdot 0 = 0$  for all  $x \in R$ . But, in general,  $0x \neq 0$  for some  $x \in R$ .
- 3. The multiplication is distributive with respect to the hyperoperation  $''+''$  on the left side, i.e.,  $x \cdot (y + z) = x \cdot y + x \cdot z$  for all  $x, y, z \in R$ .

A hypernear-ring  $R$  is called zero symmetric if  $0x = x0 = 0$  for all  $x \in R$ .

Note that for all  $x, y \in R$ , we have  $-(-x) = x$ ,  $0 = -0$ ,  $-(x + y) = -y - x$  and  $x(-y) = -xy$ .

**Definition 2.2.** [18] A two sided hyper  $R$ -subgroup of a hypernear-ring  $R$  is a subset  $H$  of  $R$  such that

- 1.  $(H, +)$  is a subhypergroup of  $(R, +)$ ,
  - (i)  $a, b \in H$  implies  $a + b \subseteq H$ ,
  - (ii)  $a \in H$  implies  $-a \in H$ ,
- 2.  $RH \subseteq H$ ,
- 3.  $HR \subseteq H$ .

If  $H$  satisfies (1) and (2), then it is called a left hyper  $R$ -subgroup of  $R$ . If  $H$  satisfies (1) and (3), then it is called a right hyper  $R$ -subgroup of  $R$ .

**Definition 2.3.** [6] Let  $(N, +, \cdot)$  be any right nearring. The triple  $(N(I), +, \cdot)$  is called a right neutrosophic nearring. For all  $x = (a, bI)$ ,  $y = (c, dI) \in N(I)$  with  $a, b, c, d \in N$ , we define:

- 1.  $x + y = (a, bI) + (c, dI) = (a + c, (b + d)I)$ .
- 2.  $-x = -(a, bI) = (-a, -bI)$ .
- 3.  $x \cdot y = (a, bI) \cdot (c, dI) = (ac, (ad + bc + bd)I)$ .

The zero element in  $(N, +)$  is represented by  $(0, 0)$  in  $(N(I), +)$ . Any element  $x \in N$  is represented by  $(x, 0)$  in  $N(I)$ .  $I$  in  $N(I)$  is sometimes represented by  $(0, I)$  in  $N(I)$ .

**Definition 2.4.** [6] Let  $(N(I), +, \cdot)$  be a right neutrosophic nearring.

- 1.  $N(I)$  is called abelian, if  $(a, bI) + (c, dI) = (c, dI) + (a, bI) \forall (a, bI), (c, dI) \in N(I)$ .
- 2.  $N(I)$  is called commutative, if  $(a, bI) \cdot (c, dI) = (c, dI) \cdot (a, bI) \forall (a, bI), (c, dI) \in N(I)$ .
- 3.  $N(I)$  is said to be distributive, if  $N(I) = N_d(I)$ , where

$$N_d(I) = \{d \in N(I) : d(m + n) = dm + dn, \forall m, n \in N(I)\}.$$

- 4.  $N(I)$  is said to be zero-symmetric, if  $N(I) = N_0(I)$ , where

$$N_0(I) = \{n \in N(I) : n0 = 0\}.$$

The following should be noted:

- (i)  $N(I)$  is abelian only if  $(N, +)$  is abelian.
- (ii)  $N(I)$  is commutative only if  $(N, \cdot)$  is commutative.
- (iii)  $N(I)$  is distributive only if  $N$  is distributive.
- (iv)  $N(I)$  is zero-symmetric only if  $N$  is zero-symmetric

### 3 Development of neutrosophic hypernear-rings

In this section, we develop the concept of neutrosophic hypernear-rings and present some of their basic properties.

**Definition 3.1.** Let  $(R, +, \cdot)$  be any hypernear-ring. The triple  $(R(I), +', \odot)$  is a neutrosophic hypernear-ring generated by  $R$  and  $I$ , where  $+'$  and  $\odot$  are hyperoperations.

For all  $r_1 = (u, vI), r_2 = (s, tI) \in R(I)$  with  $u, v, s, t \in R$ , we define :

1.  $r_1 +' r_2 = (u, vI) +' (s, tI) = \{(p, qI) : p \in u + s, q \in v + t\}$  for all  $r_1, r_2 \in R$ ,
2.  $r_1 \odot r_2 = (u, vI) \odot (s, tI) = (u \odot s, (u \odot t + v \odot s + v \odot t)I)$ , the  $"\odot"$  and  $"+"$  on the right are respectively the ordinary multiplication and hyperaddition in  $R$ ,
3.  $-r_1 = -(u, vI) = (-u, -vI)$ .

We represent element  $x \in R$  by  $(x, 0I) \in R(I)$ , and  $0 \in (R, +)$  by  $(0, 0I) \in (R(I), +')$ .  $I \in R(I)$  may also be written as  $(0, I)$ .

**Lemma 3.2.** Let  $(R(I), +', \odot)$  be any neutrosophic hypernear-ring. Let  $r_1 = (u, vI), r_2 = (s, tI) \in R(I)$  with  $u, v, s, t \in R$ . For all  $r_1, r_2 \in R(I)$  we have

1.  $-(-r_1) = -(-a, -bI) = (-(-a), -(-b)I) = (a, bI)$ ,
2.  $-(r_1 +' r_2) = -r_1 - r_2$ ,
3.  $-(0, 0I) = (0, 0I)$ ,
4.  $r_1 \odot (-r_2) = -(r_1 \odot r_2)$ .

*Proof.* The proof is similar to the proof in classical case. □

**Definition 3.3.** Let  $(R(I), +', \odot)$  be a neutrosophic hypernear-ring. An element  $(x, yI) \in R(I)$  is said to be idempotent if  $(x, yI)^2 = (x, yI)$ .

**Definition 3.4.** Let  $(R(I), +', \odot)$  be a neutrosophic hypernear-ring.

1.  $R(I)$  is called zero-symmetric neutrosophic hypernear-ring, if  $R(I) = R_{(0,0I)}(I)$ , where

$$R_{(0,0I)}(I) = \{(x, yI) \in R(I) \mid (0, 0I) \odot (x, yI) = (0, 0I)\}.$$

2.  $R(I)$  is called a constant neutrosophic hypernear-ring, if  $R(I) = R_c(I)$  where

$$R_c(I) = \{(x, yI) \in R(I) \mid (x, yI) \odot (p, qI) = (p, qI), \forall (p, qI) \in R(I)\}.$$

**Proposition 3.5.** Every neutrosophic hypernear-ring is a hypernear-ring.

*Proof.* Let  $(R(I), +', \odot)$  be a neutrosophic hypernear-ring.

1. We shall show that  $(R(I), +')$  is a quasi canonical hypergroup.

(a) Let  $(a, bI), (c, dI), (e, fI) \in R(I)$ . Then

$$\begin{aligned} ((a, bI) +' (c, dI)) +' (e, fI) &= \{(x, yI) : x \in a + c, y \in b + d\} +' (e, fI) \\ &= \{(p, qI) : p \in x + e, q \in y + f\} \\ &= \{(p, qI) : p \in (a + c) + e, q \in (b + d) + f\} \\ &= \{(p, qI) : p \in a + (c + e), q \in b + (d + f)\} \\ &= \{(p, qI) : p \in a + u, q \in b + v\} \\ &= (a, bI) +' \{(u, vI) : u \in c + e, v \in d + f\} \\ &= (a, bI) +' ((c, dI) +' (e, fI)). \end{aligned}$$

(b) Let  $(0, 0I) \in R(I)$ , then for all  $(a, bI) \in R(I)$  we have

$$\begin{aligned} (a, bI) +' (0, 0I) &= \{(x, yI) : x \in a + 0, y \in b + 0\} \\ &= \{(x, y) : x \in a, y \in b\} \\ &= \{(a, bI)\}. \end{aligned}$$

Following similar approach we can show that  $(0, 0I) + (a, bI) = \{(a, bI)\}$ . Hence there exists a neutral element in  $R(I)$ .

(c) Let  $(a, bI), -(a, bI) \in R(I)$ , then

$$\begin{aligned}(a, bI) + ' (-a, bI) &= (a, bI) + ' (-a, -bI) \\ &= \{(x, yI) : x \in a + (-a), y \in b + (-b)\} \\ &= \{(x, yI) : x \in (-a) + a, y \in (-b) + b\} \\ &= \{(x, yI) : x \in \{0\}, y \in \{0\}\} \\ &\implies (0, 0I) \in (a, bI) + ' (-a, bI).\end{aligned}$$

Hence  $-(a, bI)$  is the unique inverse of any  $(a, bI) \in R(I)$ .

(d) Suppose that  $(x, yI) \in (a, bI) + ' (c, dI)$  then

$$\begin{aligned}(x, yI) &\in \{(p, qI) : p \in a + c, q \in b + d\} \\ &= \{(p, qI) : c \in -a + p, d \in -b + q\} \\ &= \{(c, dI) : c \in -a + p, d \in -b + q\} \\ &\implies (c, dI) \in -(a, bI) + ' (x, yI).\end{aligned}$$

Following the approach above we can also establish that  $(a, bI) \in (x, yI) - (c, dI)$ .

2. Let  $(a, bI), (c, dI), (e, fI) \in R(I)$  then

(a)  $(a, bI) \odot (c, dI) = (p, qI) \in R(I)$ ,  $p = ac$  and  $q \in (ad + bc + bd)$ .

(b) Let  $(a, bI), (c, dI), (e, fI) \in R(I)$  then

$$\begin{aligned}((a, bI) \odot (c, dI)) \odot (e, fI) &= (ac, (ad + bc + bd)I) \odot (e, fI) \\ &= ((ac)e, ((ac)f + (ad)e + (bc)e + (bd)e + (ad)f + (bc)f + (bd)f)I) \\ &= (a(ce), (a(cf) + a(de) + b(ce) + b(de) + a(df) + b(cf) + b(df))I) \\ &= (a(ce), (a(cf) + a(de) + a(df) + b(ce) + b(cf) + b(de) + b(df))I) \\ &= (a, bI) \odot ((c, dI) \odot (e, fI)).\end{aligned}$$

$$\text{And } (a, bI) \odot (0, 0I) = (a0, (a0 + b0 + b0)I) = (0, 0I).$$

3. Now, it remains to show the distributive of  $(\odot)$  with respect to  $(+')$  on the left side.

Let  $(a, bI), (c, dI), (e, fI) \in R(I)$  then

$$\begin{aligned}(a, bI) \odot ((c, dI) + ' (e, fI)) &= (a, bI) \odot \{(x, yI) : x \in c + e, y \in d + f\} \\ &= \{(a, bI) \odot (x, yI) : x \in c + e, y \in d + f\} \\ &= (ax, (ax + ay + bx + by)I) \\ &= (a(c + e), (a(c + e) + a(d + f) + b(c + e) + b(d + f))I) \\ &= (ac + ae, q \in (ac + ae + ad + af + bc + be + bd + bf)I) \\ &= (ac, (ac + ad + bc + bd)I) + ' (ae, q_2 \in (ae + af + be + bf)I) \\ &= ((a, bI) \odot (c, dI)) + ' ((a, bI) \odot (e, fI)).\end{aligned}$$

□

**Proposition 3.6.** Let  $\{R_i(I)\}_i^n$  be a family of neutrosophic hypernear-rings. Then  $(\prod_{i=1}^n R_i(I), +', \odot)$  is a neutrosophic hypernear-ring.

*Proof.* 1. Let  $(a_i, b_iI), (c_i, d_iI) \in \prod_{i=1}^n R_i(I)$ , with  $a_i, b_i, c_i, d_i \in R_i$  for  $i = 1 \cdots n$ .

Following the approach in 1 of Proposition 3.5 we have that  $(\prod_{i=1}^n R_i(I), +')$  is a neutrosophic quasi canonical hypernear-ring.

2. Let  $(a_i, b_iI), (c_i, d_iI) \in \prod_{i=1}^n R_i(I)$ , with  $a_i, b_i, c_i, d_i \in R_i$  for  $i = 1 \cdots n$ .

Following the approach in 2 of Proposition 3.5 we have that  $(\prod_{i=1}^n R_i(I), \odot)$  is a neutrosophic semihypergroup.

3. To show that  $(\odot)$  is distributive with respect to  $(+')$  on the left side, we follow the approach in 3 of Proposition 3.5

Hence we have that  $(\prod_{i=1}^n R_i(I), +', \odot)$  is a neutrosophic hypernear-ring.

□

**Proposition 3.7.** Let  $M(I)$  be neutrosophic hypernear-rings and  $N$  be a hypernear-ring. Then  $M(I) \times N$  is a neutrosophic hypernear-ring.

*Proof.* The proof follows from Proposition 3.6. □

**Example 3.8.** Let  $(R(I), +)$  be a neutrosophic hypergroup and let  $M_{(0,0I)}^{R(I)}$  be defined by

$$M_{(0,0I)}^{R(I)} = \{f : R(I) \longrightarrow R(I)\},$$

such that  $f((0, 0I)) = (0, 0I)$ . For all  $f, g \in M_{(0,0I)}^{R(I)}$  we define the hyperoperation  $f +' g$  of mappings as follows:

$$(f +' g)((x, yI)) = \{h \in M_{(0,0I)}^{R(I)} \mid \forall (x, yI) \in R(I), h((x, yI)) \in f((x, yI)) + g((x, yI))\}.$$

$$f \circ g((x, yI)) = f(g((x, yI))).$$

Here the  $\circ$  on the right is the hyperoperation in  $(R, +)$  and  $\circ$  is a composition of functions.

Then  $(M_{(0,0I)}^{R(I)}, +', \circ)$  is a zero-symmetric neutrosophic hypernear-ring.

Let  $f, g \in M_{(0,0I)}^{R(I)}$  and  $(x, yI) \in R(I)$ . Since  $f(x, yI) +' g(x, yI) \neq \emptyset$ , then there exists  $h \in M_{(0,0I)}^{R(I)}$  such that  $h((x, yI)) \in f((x, yI)) +' g((x, yI))$ . Obviously,  $h((0, 0I)) \in f((0, 0I)) +' g((0, 0I)) = \{(0, 0I)\}$ , i.e.,  $h((0, 0I)) = (0, 0I)$ .

Now we shall show that  $(M_{(0,0I)}^{R(I)}, +')$  is a neutrosophic quasi canonical hypergroup.

1. Let  $f, g, h \in M_{(0,0I)}^{R(I)}$  and  $(x, yI) \in R(I)$  then

$$\begin{aligned} (f +' g) +' h &= \{p \mid \forall (x, yI) \in R(I), p((x, yI)) \in f((x, yI)) + g((x, yI))\} +' h \\ &= \{q \mid \forall (x, yI) \in R(I), q((x, yI)) \in p((x, yI)) + h((x, yI))\} \\ &= \{u \mid \forall (x, yI) \in R(I), u((x, yI)) \in (f((x, yI)) + g((x, yI))) + h((x, yI))\} \\ &= \{u \mid \forall (x, yI) \in R(I), u((x, yI)) \in f((x, yI)) + g((x, yI)) + h((x, yI))\} \\ &= \{u \mid \forall (x, yI) \in R(I), u((x, yI)) \in f((x, yI)) + (g((x, yI)) + h((x, yI)))\} \\ &= \{s \mid \forall (x, yI) \in R(I), s((x, yI)) \in f((x, yI)) + t((x, yI))\} \\ &= f +' \{t \mid \forall (x, yI) \in R(I), t((x, yI)) \in g((x, yI)) + h((x, yI))\} \\ &= f +' (g +' h). \end{aligned}$$

2. Let  $\tau \in M_{(0,0I)}^{R(I)}$  be defined by  $\tau((x, yI)) = (0, 0I)$ , then for all  $f \in M_{(0,0I)}^{R(I)}$  we have

$$\begin{aligned} (f +' \tau)((x, yI)) &= \{g \mid \forall (x, yI) \in R(I), g \in f((x, yI)) + \tau((x, yI))\} \\ &= \{g \mid \forall (x, yI) \in R(I), g \in f((x, yI)) + (0, 0I)\} \\ &= \{f((x, yI))\}. \end{aligned}$$

Similarly, it can be shown that  $(\tau +' f)((x, yI)) = \{f((x, yI))\}$ . Hence, there exists a neutral function  $\tau \in M_{(0,0I)}^{R(I)}$ .

3. Let  $f, -f \in M_{(0,0I)}^{R(I)}$  with  $(-f)((x, yI)) = -f((x, yI))$  then

$$\begin{aligned} (f +' (-f))((x, yI)) &= \{g \mid \forall (x, yI) \in R(I), g \in f((x, yI)) + (-f((x, yI)))\} \\ &= \{g \mid \forall (x, yI) \in R(I), g \in f((x, yI)) - f((x, yI))\} \\ &= \{g \mid \forall (x, yI) \in R(I), g \in \{0\}\} \end{aligned}$$

$\therefore \tau((x, yI)) \in f((x, yI)) - f((x, yI)) \implies (0, 0I) \in f((x, yI)) - f((x, yI))$ .

Hence  $-f$  is the unique inverse of any  $f \in M_{(0,0I)}^{R(I)}$ .

4. Suppose that  $h \in f +' g$ . Then

$$\begin{aligned} h &\in \{p \mid \forall (x, yI) \in R(I) p((x, yI)) \in f((x, yI)) + g((x, yI))\} \\ &= \{p \mid \forall (x, yI) \in R(I) g((x, yI)) \in -f((x, yI)) + p((x, yI))\} \\ &= \{g \mid \forall (x, yI) \in R(I) g((x, yI)) \in -f((x, yI)) + p((x, yI))\}. \end{aligned}$$

Then we have that  $g \in (-f) +' h$ . Similarly, it can be shown that  $f \in h +' (-g)$ . Hence  $h \in f +' g$  implies that  $f \in h +' (-g)$  and  $g \in (-f) +' h$ .

Accordingly,  $(M_{(0,0I)}^{R(I)}, +')$  is a neutrosophic quasicanonical hypergroup.

It can easily be established that  $(M^{R(I)}, \circ)$  is a semihypergroup having  $\tau$  as a bilaterally absorbing element such that  $(f \circ \tau)(x, yI) = f(\tau(x, yI)) = f((0, 0I)) = (0, 0I) = \tau((x, yI))$  i.e.,  $f \circ \tau = f\tau = \tau$ .

So, it remains to prove that the operation  $\circ$  is distributive with respect to the hyperoperation on the left side.

Let  $f, g, h \in M_{(0,0I)}^{R(I)}$  then

$$\begin{aligned} f \circ (g +' h) &= f \circ \{t \mid \forall (x, yI) \in R(I), t((x, yI)) \in g((x, yI)) + h((x, yI))\} \\ &= \{p \mid \forall (x, yI) \in R(I), p((x, yI)) \in ft((x, yI))\} \\ &= \{p \mid \forall (x, yI) \in R(I), p((x, yI)) \in fg((x, yI)) + fh((x, yI))\} \\ &= f \circ g((x, yI)) +' f \circ h((x, yI)). \end{aligned}$$

Then it follows that  $(M_{(0,0I)}^{R(I)}, \circ, +')$  is a zero-symmetric neutrosophic hypernear-ring.

**Example 3.9.** Let  $R(I) = \{x_0 = (0, 0I), x_1 = (a, 0I), x_2 = (b, 0I), x_3 = (0, aI), x_4 = (0, bI), y_1 = (a, aI), y_2 = (a, bI), y_3 = (b, aI), y_4 = (b, bI)\}$  be a neutrosophic set. Let  $N_x = \{x_0, x_1, x_2, x_3, x_4\}$  and  $N_y = \{y_1, y_2, y_3, y_4\}$ . Define hyperoperations  $+'$  and  $''\odot''$  on  $R(I)$  as in the table below.

Table 1: (i) Cayley table for the hyper operation  $+'$  and (ii) Cayley table for the hyper operation  $''\odot''$

(i)

$+'$	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$y_1$	$y_2$	$y_3$	$y_4$
$x_0$	$\{x_0\}$	$\{x_1\}$	$\{x_2\}$	$\{x_3\}$	$\{x_4\}$	$\{y_1\}$	$\{y_2\}$	$\{y_3\}$	$\{y_4\}$
$x_1$	$\{x_1\}$	$\left\{ \begin{matrix} x_0, \\ x_1, \\ x_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ x_2 \end{matrix} \right\}$	$\{y_1\}$	$\{y_2\}$	$\left\{ \begin{matrix} x_3, \\ y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_4, \\ y_2, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_2 \\ y_4 \end{matrix} \right\}$
$x_2$	$\{x_2\}$	$\left\{ \begin{matrix} x_1, \\ x_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_0, \\ x_1, \\ x_2 \end{matrix} \right\}$	$\{y_3\}$	$\{y_4\}$	$\left\{ \begin{matrix} y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_2, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_4 \\ y_2 \\ y_4 \end{matrix} \right\}$
$x_3$	$\{x_3\}$	$\{y_1\}$	$\{y_3\}$	$\left\{ \begin{matrix} x_0, \\ x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_3 \\ y_4 \end{matrix} \right\}$
$x_4$	$\{x_4\}$	$\{y_2\}$	$\{y_4\}$	$\left\{ \begin{matrix} x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_0, \\ x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$
$y_1$	$\{y_1\}$	$\left\{ \begin{matrix} x_3, \\ y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_1, \\ y_2 \end{matrix} \right\}$	$R(I)$	$\left\{ \begin{matrix} x_3, \\ x_4, \\ N_y \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ x_2, \\ N_y \end{matrix} \right\}$	$\{N_y\}$
$y_2$	$\{y_2\}$	$\left\{ \begin{matrix} x_4, \\ y_2, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_2, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4, \\ N_y \end{matrix} \right\}$	$R(I)$	$\{N_y\}$	$\left\{ \begin{matrix} x_1, \\ x_2, \\ N_y \end{matrix} \right\}$
$y_3$	$\{y_3\}$	$\left\{ \begin{matrix} y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ y_1, \\ y_3 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ x_2, \\ N_y \end{matrix} \right\}$	$\{N_y\}$	$R(I)$	$\left\{ \begin{matrix} x_3, \\ x_4, \\ N_y \end{matrix} \right\}$
$y_4$	$\{y_4\}$	$\left\{ \begin{matrix} y_2, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_4, \\ y_2, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$	$\{N_y\}$	$\left\{ \begin{matrix} x_1, \\ x_2, \\ N_y \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4, \\ N_y \end{matrix} \right\}$	$R(I)$

(ii)

$\odot$	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$y_1$	$y_2$	$y_3$	$y_4$
$x_0$	$x_0$	$x_0$	$x_0$	$x_0$	$x_0$	$x_0$	$x_0$	$x_0$	$x_0$
$x_1$	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$	$y_1$	$y_2$	$y_3$	$y_4$
$x_2$	$x_0$	$x_2$	$x_1$	$x_4$	$x_3$	$y_4$	$y_3$	$y_2$	$y_1$
$x_3$	$x_0$	$x_3$	$x_4$	$x_3$	$x_4$	$\left\{ \begin{matrix} x_0, \\ x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_0, \\ x_3, \\ x_4 \end{matrix} \right\}$
$x_4$	$x_0$	$x_4$	$x_3$	$x_4$	$x_3$	$\left\{ \begin{matrix} x_0, \\ x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_0, \\ x_3, \\ x_4 \end{matrix} \right\}$
$y_1$	$x_0$	$y_1$	$y_4$	$\left\{ \begin{matrix} x_0, \\ x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_0, \\ x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1 \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1 \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$
$y_2$	$x_0$	$y_2$	$y_3$	$\left\{ \begin{matrix} x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1 \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1 \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3, \\ y_4 \end{matrix} \right\}$
$y_3$	$x_0$	$y_3$	$y_2$	$\left\{ \begin{matrix} x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_3 \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3 \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2 \\ y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$
$y_4$	$x_0$	$y_4$	$y_1$	$\left\{ \begin{matrix} x_0, \\ x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_0, \\ x_3, \\ x_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2, \\ y_3 \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_2 \\ y_3, \\ y_4 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$	$\left\{ \begin{matrix} x_1, \\ y_1, \\ y_2 \end{matrix} \right\}$

Then  $(R(I), +', \odot)$  is neutrosophic hypernear-ring .

**Proposition 3.10.** Let  $R(I)$  be a neutrosophic hypernear-ring.  $R(I)$  is zero-symmetric only if  $R$  is zero-symmetric hypernear-ring.

*Proof.* Let  $R(I)$  be a neutrosophic hypernear-ring and let  $R$  be a zero symmetric hypernear-ring. Then for all  $(x, yI) \in R(I)$  and for  $(0, 0I) \in R(I)$  we have

$$\begin{aligned} (0, 0I) \odot (x, yI) &= (0x, (0y + 0x + 0y)I) \\ &= (0, 0I). \end{aligned}$$

Hence,  $R(I)$  is zero-symmetric neutrosophic hypernear-ring.  $\square$

**Proposition 3.11.** Let  $R(I)$  be a neutrosophic hypernear-ring and let  $R$  be a constant hypernear-ring. Then generally,  $R(I)$  is not a constant hypernear-ring.

*Proof.* Let  $R(I)$  be a neutrosophic hypernear-ring and let  $R$  be a constant hypernear-ring. Then, for all  $(a, bI), (x, yI) \in R(I)$  we have,

$$\begin{aligned} (a, bI) \odot (x, yI) &= (ax, (ay + bx + by)I) \\ &= (x, (y + x + y)I) \quad \because R \text{ is constant.} \\ &\neq (x, yI). \end{aligned}$$

$\square$

**Remark 3.12.**  $R(I) = \{(x, yI) : x, y \in R\}$  will be a constant hypernear-ring if  $x$  is the zero element in the constant hypernear-ring  $R$ . And, for each  $z \in R$ ,  $z + z = z$ , and  $0z = z$ .

To see this, pick any  $(0, bI), (0, aI) \in R(I)$ . Since  $z + z = z$  and  $0z = z$  for all  $z \in R$  we have

$$\begin{aligned} (0, bI) \odot (0, aI) &= (0, (0a + b0 + ba)I) \\ &= (0, (a + 0 + a)I) \\ &= (0, (a + a)I) \\ &= (0, aI). \end{aligned}$$

**Proposition 3.13.** Every element in a constant neutrosophic hypernear-ring is idempotent.

*Proof.* The proof follows easily from definition of a constant neutrosophic hypernear-ring.  $\square$

**Definition 3.14.** Let  $R(I)$  be a neutrosophic hypernear-ring and let  $N(I)$  be a nonempty subset of  $R(I)$ .  $N(I)$  is called a neutrosophic subhypernear-ring if  $N(I)$  is a neutrosophic hypernear-ring and  $N(I)$  contains a proper subset which is a subhypernear-ring of  $R$ .

**Proposition 3.15.** Let  $R(I)$  be a neutrosophic hypernear-ring. The neutrosophic subset

$$M(I)_{(0,0I)} = \{(x, yI) \in R(I) : (0, 0I) \odot (x, yI) = (0, 0I)\}$$

of  $R(I)$  is a zero-symmetric neutrosophic subhypernear-ring of  $R(I)$ .

*Proof.* Let  $(a, bI), (c, dI) \in M(I)_{(0,0I)}$ , then  $(0, 0I) \odot (a, bI) = (0, 0I)$  and  $(0, 0I) \odot (c, dI) = (0, 0I)$ .

1. Since every element in  $M(I)_{(0,0I)}$  is of the form  $(a, bI)$ , with  $a, b \in R$ .  $M(I)_{(0,0I)}$  can be written as  $(M_0, M_0(I))$ . Here  $M_0$  is a zero-symmetry subhypernear-ring of  $R$ . Therefore, we can conclude that  $M(I)_{(0,0I)}$  contains a proper subset which is a zero-symmetric subhypernear-ring of  $R$ .

2. We shall show that  $(M(I)_{(0,0I)}, +')$  is a zero-symmetric neutrosophic subhypergroup.

$$\begin{aligned} (0, 0I) \odot [(a, bI) +' (c, dI)] &= (0, 0I) \odot \{(p, qI) : p \in a + c, q \in b + d\} \\ &= (0, 0I) \odot (p, qI) \\ &= (0p, (0q + 0p + 0q)I) \\ &= (0, 0I). \end{aligned}$$

This shows that  $(a, bI) + (c, dI) \subseteq M(I)_{(0,0I)}$ .

And, for all  $(a, bI) \in M(I)_{(0,0I)}$ ,

$$\begin{aligned} (0, 0I) \odot -(a, bI) &= -((0, 0I) \odot (a, bI)) \quad \text{by Lemma 3.2} \\ &= -(0a, (0b + 0a + 0b)I) \\ &= -(0, 0I) = (0, 0I) \quad \text{by Lemma 3.2}. \end{aligned}$$

This shows that  $-(a, bI) \in M(I)_{(0,0I)}$ .

Thus,  $(M(I)_{(0,0I)}, +')$  is a zero-symmetric neutrosophic subhypergroup.

3. We shall show that  $(M(I)_{(0,0I)}, \odot)$  is a zero-symmetric neutrosophic subsemihypergroup.

$$\begin{aligned} (0, 0I) \odot [(a, bI) \odot (c, dI)] &= (0, 0I) \cdot [(ac, (ad + bc + bd)I)] \\ &= (0(ac), (0(ad) + 0(bc) + 0(bd) + 0(ac) + 0(ad) + 0(bc) + 0(bd))I) \\ &= ((0a)c, ((0a)d + (0b)c + (0b)d + (0a)c + (0a)d + (0b)c + (0b)d)I) \\ &= (0c, (0d + 0c + 0d + 0c + 0d + 0c + 0d)I) \\ &= (0, 0I). \end{aligned}$$

This shows that  $(a, bI) \odot (c, dI) \in M(I)_{(0,0I)}$ .

Thus  $(M(I)_{(0,0I)}, \odot)$  is a zero-symmetric neutrosophic subsemihypergroup.

Hence, we can conclude that  $(M(I)_{(0,0I)}, +', \odot)$  is a zero-symmetric neutrosophic subhypernear-ring of  $R(I)$ .  $\square$

**Definition 3.16.** Let  $(A, +, \cdot)$  be any hypernear-ring and let  $(M(I), +')$  be neutrosophic hypergroup. Suppose that

$$\psi : A \times M(I) \longrightarrow M(I)$$

is an action of  $A$  on  $M(I)$  defined by  $a \cdot (x, yI) = (ax, ayI)$ , for all  $a \in A$  and  $x, y \in M$ .

$M(I)$  is called a neutrosophic A-hypergroup, for all  $a, b \in A$  and  $(x, yI) \in M(I)$ , the following conditions hold:

1.  $(a + b)(x, yI) = a(x, yI) +' b(x, yI)$ .
2.  $(a \cdot b) \cdot (x, yI) = a(b(x, yI))$ .
3.  $a \cdot I = aI$ .
4.  $0 \cdot (x, yI) = (0, 0I)$  and  $a \cdot (0, 0I) = (0, 0I)$  for all  $(x, yI) \in M(I)$  and  $a \in A$ .

If we replace  $A$  with a neutrosophic hypernear-ring  $A(I)$ , then  $M(I)$  becomes a neutrosophic  $A(I)$ -hypergroup.

**Proposition 3.17.** Every neutrosophic A-hypergroup is an A-hypergroup.

*Proof.* Suppose that  $M(I)$  is a neutrosophic A-hypergroup. Then  $(M(I), +')$  is a hypergroup. The required result follows.  $\square$

**Definition 3.18.** Let  $S(I)$  be a nonempty subset of a neutrosophic A-hypergroup  $M(I)$ .  $S(I)$  is said to be a two-sided neutrosophic A-subhypergroup of  $M(I)$  if

1.  $(S(I), +')$  is a neutrosophic subhypergroup of  $(M(I), +')$ ,
2.  $AS(I) \subseteq S(I)$  and
3.  $S(I)A \subseteq S(I)$ .

$S(I)$  is said to be left neutrosophic A-subhypergroup if 1 and 2 are met. And  $(I)$  is called right neutrosophic A-subhypergroup if properties 1 and 3 are satisfied.

**Example 3.19.** Let  $M(I)$  be a neutrosophic A-hypergroup and  $(x, yI) \in M(I)$ , then the set

$$A(x, yI) = \{a(x, yI) : a \in A\}$$

is a left neutrosophic A-subhypergroup of  $M(I)$ .

To see this, let  $u, v \in A(x, yI)$ . Then there exist  $a_1, a_2 \in A$  such that  $u = a_1(x, yI)$  and  $v = a_2(x, yI)$  so that

$$\begin{aligned} u +' v &= a_1(x, yI) +' a_2(x, yI) = (a_1x, a_1yI) +' (a_2x, a_2yI) \\ &= \{(p, qI) : p \in a_1x + a_2x, q \in a_1y + a_2y\} \\ &= \{(p, qI) : p \in (a_1 + a_2)x, q \in (a_1 + a_2)y\} \\ &= ((a_1 + a_2)x, (a_1 + a_2)yI) \\ &= (a_1 + a_2)(x, yI) \\ &\subseteq A(x, yI) \quad \because a_1 + a_2 \in A. \end{aligned}$$

Also, for any  $u \in A(x, yI)$ , there exists  $a_1 \in A$  such that  $u = a_1(x, yI)$ .

Then  $-u = -(a_1(x, yI)) = -a_1(x, yI)$ , this implies that  $-u \in A(x, yI)$ , since  $-a_1 \in A$ .

Since,  $A(x, yI)$  can be written as  $(Ax, Ay(I))$ ,  $A(x, yI)$  contains a proper subset which is a subhypergroup. Hence,  $A(x, yI)$  is a neutrosophic subhypergroup.

Now, it remains to show that  $AA(x, yI) \subseteq A(x, yI)$ . Let  $a \in A$  and  $u = a_1(x, yI) \in A(x, yI)$ .

We have that

$$au = a(a_1(x, yI)) = (aa_1)(x, yI) \in A(x, yI), \text{ since } aa_1 \in A.$$

Hence, the set  $A(x, yI) = \{a(x, yI) : a \in A\}$  is a left neutrosophic A-subhypergroup of  $M(I)$ .

**Proposition 3.20.** Let  $B(I)$  and  $D(I)$  be two left neutrosophic A-subhypergroups of a neutrosophic A-hypergroup  $M(I)$  and let  $B_i(I)_{i \in \Lambda}$  be a family of left neutrosophic A-subhypergroups of an A-hypergroup  $M(I)$ . Then,

1.  $B(I) + D(I)$  is a left neutrosophic A-subhypergroup of  $M(I)$ .
2.  $\bigcap_{i \in \Lambda} B_i(I)$  is also a left neutrosophic A-subhypergroups of  $M(I)$ .

*Proof.* 1. We can easily show that  $B(I) + D(I)$  is a neutrosophic subhypergroup of  $M(I)$ .

Now it remain to show that  $A(B(I) + D(I)) \subseteq B(I) + D(I)$ .

Since  $B(I)$  and  $D(I)$  are left neutrosophic A-subhypergroup of  $M(I)$ ,  $AB(I) \subseteq B(I)$  and  $AD(I) \subseteq D(I)$ .

So, for  $a \in A$ ,  $(b_1, b_2I) \in B(I)$  and  $(d_1, d_2I) \in D(I)$  we have

$$\begin{aligned} A(B(I) + D(I)) &= a((b_1, b_2I) + (d_1, d_2I)) \\ &= a\{(p, qI) : p \in b_1 + d_1, q \in b_2 + d_2\} \\ &= \{(ap, aqI) : ap \in ab_1 + ad_1, aq \in ab_2 + ad_2\} \\ &= \{(u, vI) : u \in ab_1 + ad_1, v \in ab_2 + ad_2\} \\ &= (u, vI) \in AB(I) + AD(I) \\ &\subseteq B(I) + D(I). \end{aligned}$$

Hence,  $B(I) + D(I)$  is a left neutrosophic A-subhypergroup.

2. Let  $b_1 = (x, yI), b_2 = (u, vI) \in \bigcap_{i \in \Lambda} B_i(I)$  and  $a \in A$ . Then for all  $i \in \Lambda$ ,  $b_1, b_2 \in B_i(I)$ .

Since each  $B_i(I)$  is a left neutrosophic A-subhyperspace, for all  $i \in \Lambda$ ,  $b_1 - b_2 = b_1 + (-b_2) \subseteq B_i(I)$ ,  $B_i(I)$  contains a proper subset which is a subhypergroup and  $Ab_1 \subseteq B_i(I)$ .

Thus,

$$b_1 - b_2 = b_1 + (-b_2) \subseteq \bigcap_{i \in \Lambda} B_i(I),$$

$$\bigcap_{i \in \Lambda} B_i(I) \text{ contains } \bigcap_{i \in \Lambda} B_i \text{ which is a subhyperspace,}$$

$$\text{and } Ab_1 \subseteq \bigcap_{i=1} B_i(I).$$

□

**Proposition 3.21.** Let  $M(I)$  be a constant neutrosophic  $M(I)$ -hypergroup. Then

1. any neutrosophic subhypergroup of  $M(I)$  is a left neutrosophic  $M(I)$ -subhypergroup of  $M(I)$ .
2.  $M(I)$  is the only right neutrosophic  $M(I)$ -subhypergroup of  $M(I)$ .

*Proof.* 1. We know from Remark 3.12 that

$$M(I) = \{(0, mI) : m \in M\}.$$

Now, let  $N(I)$  be any neutrosophic subhypergroup of  $M(I)$ .

Let  $(0, xI) \in N(I)$  and  $(0, mI) \in M(I)$  be arbitrary. Then

$$(0, mI) \cdot (0, xI) = (0 \cdot 0, 0 \cdot x + m \cdot 0 + mxI) = (0, (x + 0 + x)I) = (0, xI) \in N(I).$$

$$\therefore M(I)N(I) \subseteq N(I).$$

Hence,  $N(I)$  a left neutrosophic  $M(I)$ -subhypergroup of  $M(I)$ .

2. First,  $(M(I), +')$  is a neutrosophic subhypergroup of  $(M(I), +')$ .  
It remains to show that  $M(I)$  is the only neutrosophic subhypergroup of  $(M(I), +')$  satisfying axiom 3 of Definition 3.18, i.e.,  $M(I)M(I) \subseteq M(I)$ .  
To see this, suppose we can find another neutrosophic subhypergroup  $U(I)$  of  $M(I)$  such that  $U(I)M(I) \subseteq U(I)$ . Then we have  $(0, 0I)M(I) = M(I) \subseteq U(I)$ , since  $M(I)$  is constant.  
So we must have that  $M(I) = U(I)$ . The proof is complete.

□

**Definition 3.22.** A neutrosophic subhypergroup  $A(I)$  of a neutrosophic hypergroup  $(R(I), +')$  is said to be normal if for all  $(x, yI) \in R(I)$ , we have  $(x, yI) + A(I) - (x, yI) \subseteq A(I)$ .

**Definition 3.23.** Let  $(R(I), +', \cdot)$  be a neutrosophic hypernear-ring and let  $A(I)$  be a normal neutrosophic subhypergroup of  $(R(I), +)$ .

1.  $A(I)$  is called a left neutrosophic hyperideal of  $R(I)$ , if for all  $(a, bI) \in A(I)$ ,  $(x, yI) \in R(I)$ , we have  $(x, yI)(a, bI) \in A(I)$ .
2.  $A(I)$  is called a right neutrosophic hyperideal of  $R(I)$ , if for all  $(x, yI), (u, vI) \in R(I)$ , we have  $((x, yI) +' A(I))(u, vI) - (x, yI)(u, vI) \subseteq A(I)$ .
3.  $A(I)$  is called a neutrosophic hyperideal of  $R(I)$ , if it is both a left and right neutrosophic hyperideal of  $R(I)$ .

**Definition 3.24.** Let  $(M(I), +')$  be a neutrosophic A-hypergroup over a hypernear-ring A. A normal neutrosophic subhypergroup  $B(I)$  of  $M(I)$  is called a neutrosophic hyperideal of  $M(I)$  if for all  $(b_1, b_2I) \in B(I)$ ,  $(x, yI) \in M(I)$  and  $a \in A$  we have

$$a \cdot ((x, yI) +' (b_1, b_2I)) - a \cdot (x, yI) \subseteq B(I).$$

**Proposition 3.25.** Let  $M(I)$  be a neutrosophic hypernear-ring. If  $U(I)$  and  $V(I)$  are any two neutrosophic hyperideals of  $M(I)$  and  $U_i(I)_{i \in \Lambda}$  is a family of neutrosophic hyperideals of  $M(I)$ , then

1.  $U(I) + V(I) = \{a | a \in u + v \text{ for some } u \in U(I), v \in V(I)\}$  is a neutrosophic hyperideal of  $M(I)$ .
2.  $U(I)V(I) = \{a | a \in \sum_{i=1}^n u_i v_i \text{ for some } u_i \in U(I), v_i \in V(I)\}$  is a neutrosophic hyperideal of  $M(I)$ .
3.  $\bigcap_{i \in \Lambda} U_i(I)$  is a neutrosophic hyperideal of  $M(I)$ .

*Proof.* The proof is the same as the proof in classical case.

□

**Proposition 3.26.** Let  $(M(I), +)$  be a neutrosophic A-hypergroup over a hypernear-ring A. Let  $B(I)$  be a neutrosophic hyperideal of  $M(I)$  and  $D(I)$  be a left neutrosophic A-subhypergroup of  $M(I)$ . Then  $D(I) + B(I)$  is a left neutrosophic A-subhypergroup of  $M(I)$ .

*Proof.* We want to show that  $A(D(I) + B(I)) \subseteq D(I) + B(I)$ .

Since  $B(I)$  is a neutrosophic hyperideal of  $M(I)$  and  $D(I)$  is a left neutrosophic A-subhypergroup of  $M(I)$ , for  $(b_1, b_2I) \in B(I)$ ,  $(x, yI) \in D(I)$  and  $a \in A$  we have

$$a \cdot ((x, yI) + (b_1, b_2I)) - a \cdot (x, yI) \subseteq B(I).$$

So, for  $(b_3, b_4I) \in B(I)$ ,

$$\begin{aligned} a \cdot ((x, yI) + (b_1, b_2I)) &= (b_3, b_4I) + a \cdot (x, yI) \\ &= (ax, (ay)I) + (b_3, b_4I) \\ &\subseteq D(I) + B(I). \end{aligned}$$

Hence,  $D(I) + B(I)$  is a left neutrosophic A-subhypergroup of  $M(I)$ .  $\square$

**Proposition 3.27.** Let  $M(I)$  be a neutrosophic A(I)-hypergroup over a neutrosophic hypernear-ring A(I). If  $B(I)$  and  $D(I)$  are any two neutrosophic hyperideals of  $M(I)$ , then

$$(B(I) : D(I)) = \{(a_1, a_2I) \in A(I) : (a_1, a_2I)D(I) \subseteq B(I)\}$$

is a neutrosophic hyperideal of  $M(I)$ .

*Proof.* Let  $(b_1, b_2I) \in (B(I) : D(I))$ ,  $(d_1, d_2I) \in D(I)$ ,  $(a_1, a_2I) \in A(I)$  and  $(m_1, m_2I) \in M(I)$  be arbitrary elements. Then  $(b_1, b_2I)(d_1, d_2I) = (b_1, b_2I)$  and it implies that  $b_1d_1 = b_1$ ,  $b_1d_2 + b_2d_1 + b_2d_2 = b_2$ . We want to show that  $(B(I) : D(I))$  is a neutrosophic hyperideal of  $M(I)$ .

To see this, we only need to show that

$$(a_1, a_2I) \cdot ((m_1, m_2I) + (b_1, b_2I)) - (a_1, a_2I)(m_1, m_2I) \subseteq (B(I) : D(I)).$$

Now,

$$\begin{aligned} &[(a_1, a_2I) \cdot ((m_1, m_2I) + (b_1, b_2I)) - (a_1, a_2I)(m_1, m_2I)](d_1, d_2I) \\ &= [(a_1, a_2I)(m_1 + b_1, (m_2 + b_2)I) - (a_1, a_2I)(m_1, m_2I)](d_1, d_2I) \\ &= [(a_1m_1 + a_1b_1, (a_1m_2 + a_1b_2 + a_2m_1 + a_2b_1 + a_2m_2 + a_2b_2)I) \\ &\quad - (a_1m_1, (a_1m_2 + a_2m_1 + a_2m_2)I)](d_1, d_2I) \\ &= [a_1b_1, (a_1b_2 + a_2b_1 + a_2b_2)I](d_1, d_2I) \\ &= ((a_1b_1)d_1, ((a_1b_1)d_2 + (a_1b_2)d_1 + (a_1b_2)d_2 + (a_2b_1)d_1 + (a_2b_1)d_2 + (a_2b_2)d_1 + (a_2b_2)d_2)I) \\ &= (a_1(b_1d_1), (a_1(b_1d_2) + a_1(b_2d_1) + a_1(b_2d_2) + a_2(b_1d_1) + a_2(b_1d_2) + a_2(b_2d_1) + a_2(b_2d_2))I) \\ &= (a_1b_1, (a_1(b_1d_2 + b_2d_1 + b_2d_2) + a_2(b_1d_2 + b_2d_1 + b_2d_2) + a_2b_1)I) \\ &= (a_1b_1, (a_1b_2 + a_2b_2 + a_2b_1)I) \\ &= (a_1, a_2I)(b_1, b_2I). \end{aligned}$$

Hence,  $(a_1, a_2I) \cdot ((m_1, m_2I) + (b_1, b_2I)) - (a_1, a_2I)(m_1, m_2I) \subseteq (B(I) : D(I))$ .  $\square$

**Definition 3.28.** Let  $M(I)$  be a neutrosophic hypernear-ring and let  $(x, yI) \in M(I)$ . The set

$$Ann((x, yI)) = \{(m, nI) \in M(I) : (x, yI)(m, nI) = (0, 0I)\}$$

is called the right annihilator of  $(x, yI)$ .

**Proposition 3.29.** Let  $M(I)$  be a zero-symmetric neutrosophic hypernear-ring. For any  $(x, yI) \in M(I)$ ,  $Ann((x, yI))$  is a right neutrosophic  $M(I)$ -subhypergroup of  $M(I)$ .

*Proof.*  $Ann((x, yI)) \neq \emptyset$ . Since there exists  $(0, 0I) \in M(I)$  such that

$$(x, yI)(0, 0I) = (x0, (x0 + y0 + y0)I) = (0, 0I).$$

Let  $(a, bI), (c, dI) \in Ann((x, yI))$ , then  $(x, yI)(a, bI) = (0, 0I)$  and  $(x, yI)(c, dI) = (0, 0I)$  from which we have  $xa = 0$ ,  $xb + ya + yb = 0$ ,  $xc = 0$  and  $xd + yc + yd = 0$ .

So,

$$\begin{aligned} (x, yI)[(a, bI) + (c, dI)] &= (x, yI)\{(p, qI) : p \in a + c, q \in b + d\} \\ &= (x, yI)(p, qI) \\ &= (xp, (xq + yp + yq)I) \\ &= (0, 0I). \end{aligned}$$

This implies that  $(a, bI) + (c, dI) \subseteq \text{Ann}((x, yI))$ .

Also,

$$\begin{aligned}(x, yI)(-(a, bI)) &= -((x, yI)(a, bI)) \quad \text{From Lemma 3.2} \\ &= -(0, 0I) \\ &= (0, 0I).\end{aligned}$$

Lastly, we will show that  $\text{Ann}((x, yI))M(I) \subseteq \text{Ann}((x, yI))$ . To this end, let  $(u, vI) \in M(I)$  and  $(a, bI) \in \text{Ann}((x, yI))$  so that

$$\begin{aligned}(x, yI)[(a, bI)(u, vI)] &= (x, yI)(au, (av + bu + bv)I) \\ &= (x(au), (x(av) + x(bu) + x(bv) + y(au) + y(av) + y(bu) + y(bv))I) \\ &= ((xa)u, ((xa)v + (xb)u + (xb)v + (ya)u + (ya)v + (yb)u + (yb)v)I) \\ &= ((xa)u, ((xb + ya + yb)u + ((xa + xb + ya + yb)v))I) \\ &= (0, 0I).\end{aligned}$$

So,  $(a, bI)(u, vI) \in \text{Ann}((x, yI))$  from which it follows that  $\text{Ann}((x, yI))M(I) \subseteq \text{Ann}((x, yI))$ .

Hence,  $\text{Ann}((x, yI))$  is a right neutrosophic  $M(I)$ -subhypergroup of  $M(I)$ .  $\square$

**Definition 3.30.** Let  $W(I)$  be a neutrosophic hyperideal of a neutrosophic hypernear-ring  $(M(I), +', \cdot)$ . The quotient  $M(I)/W(I)$  is defined by the set  $\{[m] = m + W(I) : m \in M(I)\}$ .

**Proposition 3.31.** Let  $M(I)/W(I) = \{[m] = m + W(I) : m \in V(I)\}$ .

For every  $[m] = (m_1, m_2I) + W(I)$ ,  $[n] = (n_1, n_2I) + W(I) \in M(I)/W(I)$  we define:

$$[m] \oplus [n] = (m_1, m_2I) + W(I) \oplus (n_1, n_2I) + W(I) = ((m_1 +' n_1), (m_2 +' n_2)I) + W(I)$$

and

$$[m] \odot [n] = [m \cdot n] = (m_1n_1, (m_1n_2 + m_2n_1 + m_2n_2)I) + W(I).$$

$(M(I)/W(I), \oplus, \odot)$  is a neutrosophic hypernear-ring called neutrosophic quotient hypernear-ring.

*Proof.* The proof is similar to the proof in classical case.  $\square$

**Definition 3.32.** Let  $M(I)$  and  $N(I)$  be any two neutrosophic hypernear-rings.

$$\alpha : M(I) \longrightarrow N(I)$$

is called a neutrosophic hypernear-ring homomorphism, if the following conditions hold:

1.  $\alpha$  is a hypernear-ring homomorphism,
2.  $\alpha(I) = I$ .

Note: If  $M(I)$  and  $N(I)$  are any two neutrosophic A-hypergroups. Then  $\alpha$  is called a neutrosophic A-hypergroup homomorphism if  $\alpha$  is a A-hypergroup homomorphism and  $\alpha(I) = I$ .

**Definition 3.33.** Let  $\alpha$  be a neutrosophic homomorphism from  $M(I)$  into  $N(I)$  then

1.  $\text{Ker}\alpha = \{(x, yI) \in M(I) : \alpha((x, yI)) = (0, 0I)\}$  and
2.  $\text{Im}\alpha = \{(a, bI) \in N(I) : (a, bI) = \alpha((x, yI)), (x, yI) \in M(I)\}$ .

**Proposition 3.34.** Let  $A(I)$  and  $B(I)$  be two neutrosophic A-hypergroup over a zero-symmetric hypernear-ring  $A$ . Let  $\alpha : A(I) \longrightarrow B(I)$  be a neutrosophic A-hypergroup homomorphism, then

1.  $\text{Ker}\alpha$  is not a neutrosophic hyperideal of  $A(I)$ .
2.  $\text{Ker}\alpha$  is a two-sided A-subhypergroup of  $A$ .
3.  $\text{Im}\alpha$  is a left A-neutrosophic subhypergroup of  $B(I)$ .

*Proof.* 1. Since  $\alpha$  is a neutrosophic A-hypergroup homomorphism, we know that  $\alpha(I) = I$ .

Then  $(a_1, a_2I) \in A(I)$  with  $a_1, a_2 \in A$  will be in the  $\text{Ker}\alpha$  if and only if  $a_2 = 0$ . This implies that

$$\text{ker}\alpha = \{(a_1, 0I) \in A(I)\}$$

which is just a subhypergroup of  $A$ . Hence,  $\text{ker}\alpha$  is not a neutrosophic hyperideal of  $A(I)$ .

2. From 1, we have that  $\ker\alpha = \{(a_1, 0I) \in A(I)\}$  is a subhypergroup of  $A$ . So, it remains to show that  $A(\ker\alpha) \subseteq \ker\alpha$  and  $(\ker\alpha)A \subseteq \ker\alpha$

To see this, let  $b \in A$  and  $(a_1, 0I) \in \ker\alpha$  be arbitrary then

$$b\alpha((a_1, 0I)) = \alpha((ba_1, (b0)I)) = \alpha((ba_1, 0I)) = (0, 0I) \in \ker\alpha,$$

and

$$\alpha((a_1, 0I))b = \alpha((a_1b, (0b)I)) = \alpha((a_1b, 0I)) = (0, 0I) \in \ker\alpha.$$

It implies that  $A(\ker\alpha) \subseteq \ker\alpha$  and  $(\ker\alpha)A \subseteq \ker\alpha$ . Hence  $\ker\alpha$  is a two-sided  $A$ -subhypergroup of  $A$ .

3. By definition  $Im\alpha = \{(a, bI) \in B(I) : (a, bI) = \alpha((x, yI)), (x, yI) \in A(I)\}$ . It is clear that  $Im\alpha$  is a neutrosophic subhypergroup of  $B(I)$ . So, it remains to show that  $A(Im\alpha) \subseteq (Im\alpha)$ . Now, let  $(a, bI) \in Im\alpha$  and  $a_1 \in A$  be arbitrary, where  $(a, bI) = \alpha((x, yI))$  and  $(x, yI) \in A(I)$ , then

$$a_1(a, bI) = a_1\alpha((x, yI)) = \alpha((a_1x), (a_1y)I).$$

Since  $A(I)$  is a  $A$ -hypergroup, then  $((a_1x), (a_1y)I) \in A(I)$ . So, we can say that  $\alpha((a_1x), (a_1y)I) \in Im\alpha$ . And it implies that  $A(Im\alpha) \subseteq Im\alpha$ . Hence,  $Im\alpha$  is a left neutrosophic  $A$ -subhypergroup of  $B(I)$ . □

**Proposition 3.35.** Let  $A(I)$  and  $B(I)$  be any two neutrosophic hypernear-ring and let  $\alpha : A(I) \longrightarrow B(I)$  be a neutrosophic hypernear-ring homomorphism, then

1.  $\ker\alpha$  is not a neutrosophic hyperideal of  $A(I)$ .
2.  $\ker\alpha$  is a subhypernear-ring of  $A$ .
3.  $Im\alpha$  is neutrosophic subhypernear-ring of  $B(I)$ .

*Proof.* 1. The proof follows similar approach as proof 1 of Proposition [3.34](#).

2.  $\ker\alpha = \{(a_1, 0) \in A(I)\}$ . It is easy to show that  $\ker\alpha$  is a subhypernear-ring of  $A$ .
3. The proof is similar to the proof in classical case. □

## 4 Conclusion

We investigated and presented some of the interesting results arising from the study of hypernear-rings in the neutrosophic environment. The concept of neutrosophic  $A$ -hypergroup of a hypernear-ring  $A$ , neutrosophic  $A(I)$ -hypergroup of a neutrosophic hypernear-ring  $A(I)$  and their respective neutrosophic substructures were presented. It was shown that a constant neutrosophic hypernear-ring in general is not a constant hypernear-rings. We hope to study more advanced properties of neutrosophic hypernear-ring in our future work.

## 5 Acknowledgment

The authors wish to thank the anonymous reviewers for their valuable and useful comments which have lead to the improvement of the paper.

## References

- [1] Agboola, A.A.A. and Akinleye, S.A., "Neutrosophic Hypervector Spaces", ROMAI Journal, Vol.11, pp. 1-16, 2015.
- [2] Agboola, A.A.A and Davvaz, B., "On Neutrosophic Canonical Hypergroups and Neutrosophic Hyperrings", Neutrosophic Sets and Systems. Vol.2, pp. 34-41, 2014.
- [3] Agboola, A.A.A and Davvaz, B., "Introduction to Neutrosophic Hypergroups", ROMAI J., Vol.9(2), pp. 1-10, 2013.

- [4] Agboola, A.A.A., Davvaz, B. and Smarandache, F. "Neutrosophic quadruple algebraic hyperstructures", *Ann. Fuzzy Math. Inform.* Vol.14, pp. 29-42, 2017.
- [5] Agboola, A.A.A., Ibrahim, M.A., Adeleke, E.O., Akinleye, S.A., "On Refined Neutrosophic Algebraic Hyperstructures I", *International Journal of Neutrosophic Science (IJNS)*, Vol. 5(1), pp. 29-37, 2020.
- [6] Akinleye, S.A., Adeleke, E.O. and Agboola, A.A.A. "Introduction to neutrosophic near-rings", *Ann. Fuzzy Math. Inform.* Vol.12, pp. 397-409, 2016.
- [7] Al-Tahan, M. and Davvaz, B., "Refined neutrosophic quadruple (po-)hypergroups and their fundamental group", *Neutrosophic Sets and Systems*, Vol. 27, pp. 138-153, 2019.
- [8] Corsini, P., "Hypergroupes regulier et hypermodules", Vol 20, pp. 121-135, 1975.
- [9] Corsini, P., "Prolegomena of Hypergroup Theory", Aviani Editore, Tricemo, Italian, 1993.
- [10] Corsini, P. and Leoreanu, V. "Application of Hyperstructure Theory", Vol.5 *Advances in Mathematics*. Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
- [11] Dasic, V., "Hypernear-rings, Algebraic Hyperstructures and Applications", *Proceedings of the Fourth International Congress Xanthi, Greece*, pp. 75-85, World Sci. Publ., Teaneck, NJ, 1991.
- [12] Davvaz, B. and Leoreanu-Fotea, V., "Hyperring Theory and Applications", *International Academic Press*, Palm Harbor, USA, 2007.
- [13] Gontineac, V.M., "On hypernear-rings and H-hypergroups", *Proc. Fifth Int. Congress on Algebraic Hyperstructures and Application (AHA 1993)*, Hadronic Press, Inc., USA, pp. 171-179, 1994.
- [14] Ibrahim, M.A., Agboola, A.A.A., Adeleke, E.O., Akinleye, S.A., "Introduction to Neutrosophic Subtraction Algebra and Neutrosophic Subtraction Semigroup", *International Journal of Neutrosophic Science (IJNS)*, Vol. 2(2), pp. 47-62, 2020.
- [15] Ibrahim, M.A., Agboola, A.A.A., Adeleke, E.O., Akinleye, S.A., "On Neutrosophic Quadruple Hypervector Spaces", *International Journal of Neutrosophic Science (IJNS)*, Vol. 4(1), pp. 20-35, 2020.
- [16] Ibrahim, M.A., Agboola, A.A.A., Badmus, B.S., Akinleye, S.A., "On Refined Neutrosophic Hypervector Spaces", *International Journal of Neutrosophic Science (IJNS)*, Vol. 8(1), pp. 50-71, 2020.
- [17] Ibrahim, M.A., Agboola, A.A.A., Badmus, B.S., Akinleye, S.A., "On Refined Neutrosophic Hypergroup", to appear in *International Journal of Neutrosophic Science (IJNS)*.
- [18] Kyung Ho Kim, Davvaz, B. and Eun Hwan Roh, "On Hyper R-Subgroups of Hypernear-rings", *Scientiae Mathematicae Japonicae Online*, pp. 649-656, 2007.
- [19] Mittas, J., "Hypergroupes canoniques", *Mathematica Balkanica*, Vol.2, pp.165-179, 1972.
- [20] Mittas, J., "Hyperanneaux et certaines de leurs proprietes", Vol.269, pp. A623-a626, 1969.
- [21] Massouros, C., "Some properties of certain Subhypergroups", *RATIO MATHEMATICA* Vol.25, pp.67-76, 2013.
- [22] Smarandache, F., "(T,I,F)- Neutrosophic Structures", *Neutrosophic Sets and Systems*, Vol. 8, pp. 3-10, 2015.
- [23] Stratisgopoulos, D., "Certaines classes d'hypercorps et d'hyperanneaux, in *Hypergroups, Other Multi-valued Structures and Their Applications*", University of Udine, Udine, Italy, pp. 105-110, 1985.
- [24] Vasantha Kandasamy W.B and Smarandache, F., "Basic Neutrosophic Algebraic Structures and Their Applications to Fuzzy and Neutrosophic Models", Hexis, Church Rock, 2004, <http://fs.gallup.unm.edu/eBook-otherformats.htm>
- [25] Vasantha Kandasamy, W.B. and Florentin Smarandache, "Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures", Hexis, Phoenix, Arizona, 2006, <http://www.gallup.unm.edu/smarandache/eBooks-otherformats.htm>.
- [26] Vasantha Kandasamy, W.B., "Neutrosophic Rings", Hexis, Phoenix, Arizona, 2006, <http://www.gallup.unm.edu/smarandache/eBooks-otherformats.htm>.



## A Note on Neutrosophic Polynomials and Some of Its Properties

Somen Debnath<sup>1,\*</sup> and Anjan Mukherjee<sup>2</sup>

<sup>1</sup>Department of Mathematics, Umakanta Academy, Agartala-799001; Tripura, INDIA; [somen008@rediffmail.com](mailto:somen008@rediffmail.com)

<sup>2</sup> Department of Mathematics, Tripura University, Agartala -799022; Tripura, INDIA; [mukherjee123anjan@gmail.com](mailto:mukherjee123anjan@gmail.com)

\* Correspondence: [somen008@rediffmail.com](mailto:somen008@rediffmail.com)

### Abstract

The purpose of this article is to study neutrosophic polynomials i.e. polynomials which are Neutrosophic in nature and study its properties with the help of neutrosophic numbers. Apart from this we discuss different types of neutrosophic polynomials with concrete examples and establish some theorems and results which will be useful for the further study. We also give a solution method to find the approximate roots of a neutrosophic polynomial equation.

**Keywords:** Neutrosophic numbers; Neutrosophic polynomials; Synthetic division; Multiple roots

### 1.Introduction

Zadeh [1] is the initiator of fuzzy set. It is a such kind of mathematical tool by which we can generalize the classical concepts by introducing membership function. Fuzziness is determined by a membership function. So it plays a vital role for the fuzzy representation. Fuzzy set mainly concerned with membership function and there are different methods for the determination of membership function. So sometimes to choose the best method to get the best result is a tedious job. Embedding the idea of uncertainty, Gau and Buehrer introduced vague set [2], Goguen initiated L-fuzzy set [3] and Pawlak introduced Rough set [4]. Some other extensions of fuzzy sets are mentioned in [5-7]. These generalized concepts enhance the scope to solve the uncertain problems appropriately.

Sometimes it is difficult to represent uncertainty by assigning a single real value, that's why interval-valued fuzzy sets was proposed. In expert system, in belief system etc there is a need to consider both truth-membership and the falsity membership for actual elucidation of an element in undetermined, ambivalent domain. For such situation both fuzzy set and interval-valued fuzzy set are irrelevant to use. In suchlike circumstances only intuitionistic fuzzy set is recommended. It can operate the deficient data with the help of falsity-membership. But there exists indeterminate and inconsistent data in uncertain environment which can't be handled by such type of sets.

From logical view if somebody ask me to read the mind of a person then how it can be done. We know that human mind mainly divided into three parts which are responsive mind, senseless mind and dormant mind. Dormant means it is partially responsive and partially senseless. So it can't be measured by neither membership value nor non-membership value. It is a type of indeterminacy. In real life problems also we come across with such a situation. So we need another convenient tool which can be able to measure the level of indeterminacy along with membership and non-membership. Which leads to the introduction of Neutrosophic sets. Smarandache [8] is the pioneer of Neutrosophic set. By adding indeterminacy with intuitionistic fuzzy set, Smarandache defined neutrosophic set. Pertaining the idea of Neutrosophic sets several theories, results and applications are developed by the researchers and Mathematicians of all over the world. Some of their contributions are mentioned in [9-13]. For parametric nature in a data one more numerical tool developed by Molodtsov [14]. In [15], Maji et al. study and describe various properties of soft set. It has been successfully used in various aspects. It is considered even as more general framework of modeling and manipulating the vague concept. In 2012, Maji [16] introduced Neutrosophic soft set (NSS). Application of neutrosophic soft set shown in [17]. Several works related to NSSs have been done effectively over various discipline by the neutrosophication in data.

In this work we mainly concerned with Neutrosophic polynomials and its types. We also contribute some theorems and results based on Neutrosophic polynomials and try to develop the earlier concept of classical polynomials so that we have a bright scope to use it further and to establish new theories and results related to polynomials which are indeterministic in nature. For practical purpose we include [18-21].

The present article is arranged in the following manner :

Section-2 comprised with Neutrosophic numbers and basic operations on them. In section-3, firstly defined neutrosophic polynomials and its types and then deduce some theorems and give examples of each type to justify the results. In the last section (in section-4), the main purpose of the paper is discussed briefly.

## 2. Classical Neutrosophic Numbers and basic operations

Here, we give the basic definition of Neutrosophic Numbers and its properties.

**Definition 1.** A classical Neutrosophic number is of the form  $x + y\Im$ , where  $x, y$  are real or complex coefficients and  $\Im$ =indeterminacy, such that  $0.\Im = 0$  and  $\Im^2 = \Im$ . In general  $\Im^n = \Im$ , for all positive integers  $n$ . If  $x, y$  are real then  $x + y\Im$  is called Neutrosophic real number otherwise it is known as Neutrosophic complex number.  $3 - 5\Im, 6 + 8\Im$  etc are examples of Neutrosophic real numbers and  $(4 + 3i) + (4 - 2i)\Im, (3 - 2i) + i\Im$  etc are examples of Neutrosophic complex numbers where  $i \equiv \sqrt{-1}$  and  $\Im$  denotes indeterminacy.

Generally a Neutrosophic complex number can be written as  $x + iy + z\Im + ti\Im$ , where  $x, y, z$  and  $t$  are reals and  $i \equiv \sqrt{-1}$ . Indeterminacy  $\Im$  (with non-zero coefficient) in a Neutrosophic number is known as a true Neutrosophic number. In our work we mainly concerned with Neutrosophic real numbers.

**Definition 2.** If  $p_1 + \Im q_1$  and  $p_2 + \Im q_2$  be two classical neutrosophic real numbers then their division is denoted by  $\frac{p_1 + \Im q_1}{p_2 + \Im q_2} = a + \Im b$ , where  $p_2$  and  $q_2$  are real and they never zero together and  $\Im$  denotes indeterminacy.

On comparing both sides

$$p_2 a = p_1 \text{ and } q_2 a + (p_2 + q_2)b = q_1$$

For one and only one solution, we have

$$\begin{vmatrix} p_2 & 0 \\ q_2 & p_2 + q_2 \end{vmatrix} \neq 0$$

$$\text{or, } p_2(p_2 + q_2) \neq 0$$

$$\text{or, } p_2 \neq 0 \text{ and } p_2 \neq -q_2$$

$$\text{Then, } a = \frac{p_1}{p_2} \text{ and } b = \frac{p_2 q_1 - p_1 q_2}{p_2(p_2 + q_2)}$$

$$\text{Thus, } \frac{p_1 + \Im q_1}{p_2 + \Im q_2} = \frac{p_1}{p_2} + \Im \frac{p_2 q_1 - p_1 q_2}{p_2(p_2 + q_2)}$$

**Results 1.** For the division of neutrosophic real numbers we have the following results:

1.  $\frac{p + q\Im}{p\lambda + q\lambda\Im} \frac{p + q\Im}{\lambda(p + q\Im)} = \frac{1}{\lambda}$ , for  $\lambda \neq 0$  and  $p + q\Im \neq 0$
2.  $\frac{\Im}{p + q\Im} = \frac{p}{p(p + q\Im)} \cdot \Im = \frac{1}{p + q\Im} \cdot \Im$ , for  $p \neq 0$  and  $p + q\Im \neq 0$
3.  $\frac{p + \Im q}{\lambda\Im} = \text{undefined}$ , for either  $\lambda=0$  or  $\Im$  doesnot take any non zero value.

$$\text{In particular: } \frac{\Im}{\Im} = \text{undefined}$$

4.  $\frac{p + \Im q}{\lambda} = \frac{p}{\lambda} + \frac{q}{\lambda} \cdot \Im$ , for  $\lambda \neq 0$
5.  $\frac{r}{p + q\Im} = \frac{r}{p} - \frac{qr}{p(p + q\Im)} \cdot \Im$ , for  $p \neq 0$  and  $(p + q\Im) \neq 0$

### 3. Neutrosophic Polynomials

Firstly we give the notion of neutrosophic polynomials with a concrete example then some results based on it are established.

**Definition 3.** Neutrosophic polynomial is a polynomial whose coefficients (atleast one of them contain  $\Im$ ) are neutrosophic numbers. If its coefficients are neutrosophic real numbers then it is called neutrosophic real polynomial otherwise it is called neutrosophic complex polynomial.

$P(y) = 3y^2 + (3 + \Im)y - (5 + 3\Im)$  and  $Q(x) = 4x^3 + (1 - 6i\Im)x^2 + 7\Im x - 6i\Im$  are examples of neutrosophic real polynomial and neutrosophic complex polynomial respectively.

In this work we mainly concerned with neutrosophic real polynomials.

In general any expression of the form  $P_N(x) = p_0 x^n + p_1 x^{n-1} + p_2 x^{n-2} + \dots + p_n$  where  $p_0, p_1, p_2, \dots, p_n$  are neutrosophic real numbers in which at least one of the coefficient contains indeterminacy( $\Im$ ) and  $n$  is a

positive integer is called neutrosophic real polynomial of degree  $n$  if  $p_0 \neq 0$ . The terms of a neutrosophic real polynomial with zero coefficient are normally omitted.

From logical view we give a real example of Neutrosophic polynomial in the following way:

We brought a cake from a bakery which has a volume 120 cu.units. Length of the cake is 2 units more than its breadth and its height is slightly more than its breadth. We consider  $a, b$  and  $c$  for length, breadth and height respectively. Using the given condition we write  $a = b + 2, c = b + \Im$ , where  $\Im$  is the indeterminacy involved in this problem. So such a situation can be shown as

$$120 = (b + 2)b(b + \Im)$$

$$\text{or, } (b^2 + 2b)(b + \Im) = 120$$

$$\text{or, } (b^3 + b^2\Im + 2b^2 + 2\Im b) = 120$$

$$\text{or, } b^3 + b^2(\Im + 2) + 2\Im b = 120$$

Which is the required neutrosophic polynomial for the above problem.

Now the question arises how we solve this type of equation. To do so we consider some examples.

**Example 1.** Find the roots of the neutrosophic real polynomial  $6x^2 + (10 + \Im)x + 4\Im = 0$ .

**Solution.** We have,

$$6x^2 + (10 + \Im)x + 4\Im = 0$$

$$\text{or, } x = \frac{-(10 + \Im) \pm \sqrt{(10 + \Im)^2 - 4 \cdot 6 \cdot 4\Im}}{12} \text{ (using the quadratic formula)}$$

$$\text{or, } x = \frac{-10 - \Im \pm 5\sqrt{4 - 3\Im}}{12}$$

Let,  $\sqrt{4 - 3\Im} = a + \Im b$ , where  $a$  and  $b$  are real and  $\Im \leq \frac{4}{3}$ .

$$\text{or, } 4 - 3\Im = a^2 + 2ab\Im + b^2\Im$$

$$\text{or, } 4 - 3\Im = a^2 + (2ab + b^2)\Im$$

$$a^2 = 4 \quad \left| \quad 2ab + b^2 = -3 \right.$$

$$a = \pm 2$$

$$\text{when } a = 2, b^2 + 4b + 3 = 0 \Rightarrow b = -1, -3$$

$$\text{and when } a = -2, b^2 - 4b + 3 = 0 \Rightarrow b = 3, 1$$

Putting the values of  $a$  and  $b$  in the above equation we get the values of  $x$  which are  $-\frac{\Im}{2}, \frac{-5 + \Im}{3}, \frac{-4\Im}{3}$  and  $\frac{-\Im - 10}{6}$

Therefore,  $\left\{-\frac{\Im}{2}, \frac{-5+\Im}{3}, \frac{-4\Im}{3}, \frac{-\Im-10}{6}\right\}$  is the solution set, where  $\Im \leq \frac{4}{3}$ .

Now we check the other properties of the polynomial.

$$\text{Clearly, } P_N\left(-\frac{\Im}{2}\right) = P_N\left(\frac{-5+\Im}{3}\right) = P_N\left(\frac{-4\Im}{3}\right) = P_N\left(\frac{-\Im-10}{6}\right) = 0$$

Let,  $P_N(x) = 6x^2 + (10 + \Im)x + 4\Im$  then

$$P_N(x) = 6\left[x - \left(-\frac{\Im}{2}\right)\right]\left[x - \left(\frac{-5+\Im}{3}\right)\right] = 6\left[x - \left(\frac{-4\Im}{3}\right)\right]\left[x - \left(\frac{-\Im-10}{6}\right)\right]$$

Clearly, the above neutrosophic polynomial has more than one factorization i.e it is not unique and the number of roots are more than that of the degree of the neutrosophic polynomial.

Let us take one more example to get more concrete result

**Example 2.** Find the roots of the neutrosophic polynomial  $x^2 - 6\Im = 0$

**Solution.**  $x^2 - 6\Im = 0 \Rightarrow x^2 = 6\Im \Rightarrow x = \pm\sqrt{6\Im}$

Let,  $\sqrt{6\Im} = a + \Im b$ , where  $a$  and  $b$  are real and  $\Im \geq 0$ .

$$\text{or, } 6\Im = a^2 + (2ab + b^2)\Im$$

$$\text{or, } a = 0 \mid b^2 = 6 \Rightarrow b = \pm\sqrt{6}$$

$$\text{OR, } \sqrt{6\Im} = \sqrt{6} \cdot \sqrt{\Im} = \sqrt{6} \Im$$

Therefore, the solutions are  $\pm\sqrt{6} \Im$

In this case it obeys the property of classical polynomial as the degree of the polynomial is equal to the no of roots. So there is no such proper relation between the degree and the number of roots of a neutrosophic polynomial. But one thing is clear from these two examples that number of roots will be either the degree of the polynomial or double of its degree. It is for the reader to verify the roots of the higher degree polynomial to get more concrete results.

**Remark 1.** Number of roots of a neutrosophic polynomial of degree  $n \geq 1$  will be  $n$  or  $2n$  or  $3n$ .....and so on.

**Definition4.** A neutrosophic polynomial without any zero coefficient is called a complete Neutrosophic polynomial otherwise it is incomplete.

**Definition5.** A neutrosophic polynomial whose coefficients are zero and it is represented by  $0=0.\Im$  is called vanishing Neutrosophic polynomial.

**Definition6.** Two neutrosophic polynomials are said to be equal iff their corresponding coefficients are alike and their corresponding indeterminacies converges to a fixed value.

### 3.1 Division algorithm

If  $P_N(x)$  and  $\phi_N(x)$  be two neutrosophic polynomials where  $\deg(\phi_N(x)) \leq \deg(P_N(x))$ , then to divide  $P_N(x)$  by  $\phi_N(x)$  is to establish a relation of the form

$$P_N(x) = \phi_N(x) \cdot Q_N + R_N,$$

Where  $Q_N$  and  $R_N$  are respectively the quotient and the remainder and  $\deg(R_N) < \deg(\phi_N(x))$ . Here  $Q_N$  and  $R_N$  are unique. If  $R_N = 0$ , then  $P_N(x)$  is completely divisible by  $\phi_N(x)$ .

### 3.2 Synthetic division method

It is a process in which we divide a polynomial of degree  $n$  by a binomial under neutrosophic environment.

Let,

$$P_N(x) = (a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + (a_2 + \Im)x^{n-2} + \dots + (a_n + \Im), \text{ where } (a_0 + \Im) \neq 0$$

and the binomial is  $(x - (\Im + h))$

By division algorithm, we have

$$(a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + (a_2 + \Im)x^{n-2} + \dots + (a_n + \Im) = (x - (\Im + h))[(b_0 + \Im)x^{n-1} + (b_1 + \Im)x^{n-2} + (b_2 + \Im)x^{n-3} + \dots + (b_{n-1} + \Im) + (R + \Im)]$$

Equating the coefficients of like powers, we have

$$a_0 + \Im = b_0 + \Im$$

$$a_1 + \Im = (b_1 + \Im - b_0\Im - \Im - hb_0 - \Im h) \Rightarrow b_1 = a_1 + hb_0 + \Im(1 + h + b_0)$$

Similarly,

$$b_2 = a_2 + hb_1 + \Im(1 + h + b_1)$$

$$b_3 = a_3 + hb_2 + \Im(1 + h + b_2)$$

.....

$$R = a_n + hb_{n-1} + \Im(1 + h + b_{n-1})$$

Thus, we establish an easy approach by which we calculate the coefficients of the quotient and the remainder as follows:

$$\begin{array}{ccccccc} a_0 + \Im & a_1 + \Im & a_2 + \Im & \dots & a_n + \Im & & \\ & b_0(\Im + h) + \Im(\Im + h) & b_1(\Im + h) + \Im(\Im + h) & \dots & b_{n-1}(\Im + h) + \Im(\Im + h) & & \end{array}$$

---


$$\begin{array}{ccccccc} b_0 + \Im & b_1 + \Im & b_2 + \Im & \dots & R + \Im & & \end{array}$$

**Note1.** If we divide a neutrosophic polynomial  $P_N(x)$  by  $(ax - (b + \Im))$ , we first to find the quotient( $\square$ ) and remainder( $\Re$ ) in the division of  $P_N(x)$  by  $\left(x - \frac{(b + \Im)}{a}\right)$ , then the quotient and remainder in the division of  $P_N(x)$  by  $(ax - (b + \Im))$  will be  $\frac{\square}{a}$  and  $\Re$  respectively.

**Example3.** Using synthetic division method divide  $x^4 + (5 + \Im)x^3 + (4 + \Im)x^2 + (8 + \Im)x - (20 + \Im)$  by  $(x - (1 + \Im))$

**Solution.** By synthetic division method, we have

$$\begin{array}{r|rrrrr} 1 & 5 + \Im & 4 + \Im & 8 + \Im & -(20 + \Im) & \\ & (1 + \Im) & 6 + 8\Im & 10 + 18\Im & 18 + 56\Im & \end{array}$$

---


$$\begin{array}{r|rrrrr} 1 & 6 + \Im & 10 + 9\Im & 18 + 19\Im & -2 + 55\Im & \end{array}$$

$$Q = x^3 + (6 + \Im)x^2 + (10 + 9\Im)x + (18 + 19\Im) \text{ and } R = -2 + 55\Im$$

### 3.3 Remainder theorem

**Statement.** If a Neutrosophic real polynomial be divided by a neutrosophic binomial  $(x - (\Im + h))$  then the remainder is  $P_N(\Im + h)$

**Proof.** By division algorithm,

$$P_n(x) = (x - (\Im + h))Q + (R + \Im)$$

Putting  $x = \Im + h$ , we have  $(R + \Im) = P_N(\Im + h)$

**Example4.** Find the remainder when  $x^3 - (3 + \Im)x^2 + 4x - 3$  is divided by  $(x - (2 + \Im))$

**Solution.** By remainder theorem,

$$P_n(2 + \Im) = 1 - \Im, \text{ which is the required remainder.}$$

**Example5.** Find a relation between  $r$  and  $s$  when  $2x^3 - (7 + \Im)x^2 + (r + \Im)x + (s + \Im)$  is exactly divisible by  $(x - (3 + \Im))$

**Solution.** By remainder theorem,

$$P_n(3 + \Im) = 0 \Rightarrow r(3 + \Im) + s = 9 - 14\Im, \text{ which is the required relation}$$

**Results2.** We have the following results

1. Let  $P_N(x)$  be a Neutrosophic Polynomial which vanishes when  $x$  takes the values  $p_1 + \Im$ ,  $p_2 + \Im$ ,  $p_3 + \Im$ ,  $\dots, p_n + \Im$  where no two of which are equal then the product

$$(x - (p_1 + \Im))(x - (p_2 + \Im)), \dots, (x - (p_n + \Im)) = 0.$$

2. A Neutrosophic polynomial  $P_N(x)$  of  $n$ -th degree ( $n \geq 1$ ) can be vanish for more than  $n$  values of  $x$ .

### 3.4 Method to express a given neutrosophic real polynomial $P_N(x)$ as a function of $(x - (\Im + h))$

**Procedure.**

$$\text{Let } P_N(x) = (a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + (a_2 + \Im)x^{n-2} + \dots + (a_n + \Im) \quad (1)$$

$$= b_0(x - (\Im + h))^n + b_1(x - (\Im + h))^{n-1} + \dots + b_{n-1}(x - (\Im + h)) + (b_n + \Im) \quad (2)$$

$$= (x - (\Im + h)) \left[ b_0(x - (\Im + h))^{n-1} + b_1(x - (\Im + h))^{n-2} + \dots + b_{n-1} \right] + (b_n + \Im)$$

$$= (x - (\Im + h))Q[(x - (\Im + h))] + (b_n + \Im)$$

Where  $Q[(x - (\Im + h))]$  is the quotient and  $(b_n + \Im)$  is the remainder. Therefore, the remainder is the last coefficient of (2). If  $Q[(x - (\Im + h))]$  divided by  $(x - (\Im + h))$  the remainder will be  $(b_{n-1} + \Im)$ , which is the coefficient of  $(x - (\Im + h))$  in (2).

If we continue like this, we obtain, after each successive division  $(b_n + \Im), (b_{n-1} + \Im), \dots, (b_1 + \Im)$  and  $(b_0 + \Im) = (a_0 + \Im)$ .

Thus, the synthetic division method is applied for successive division as at each stage we get the quotient and the remainder.

**Example 6.** Express  $P_N(x) = x^5 + (5 + \Im)x^3 + (3 + \Im)x$  as a polynomial of  $(x - (1 + \Im))$

**Solution.** By using synthetic division method successively, we have

**Theorem1.** In the neutrosophic polynomial  $(a_0 + \mathfrak{I})x^n + (a_1 + \mathfrak{I})x^{n-1} + (a_2 + \mathfrak{I})x^{n-2} + \dots + (a_n + \mathfrak{I})$ , where  $(a_0 + \mathfrak{I}) \neq 0$  the leading term  $(a_0 + \mathfrak{I})x^n$  exceeds the sum of the remaining terms for  $x \geq \frac{a_k + \mathfrak{I}}{a_0 + \mathfrak{I}} + 1$ , where  $a_k + \mathfrak{I}$  is the greatest coefficient irrespective of sign.

$$(a_0 + \Im)x^n > (a_1 + \Im)x^{n-1} + (a_2 + \Im)x^{n-2} + \dots + (a_n + \Im)$$

if  $(a_0 + \Im)x^n > (a_k + \Im)(x^{n-1} + x^{n-2} + \dots + (x+1))$ ,  $(a_k + \Im)$  being the greatest coefficient

$$\text{i.e., if } (a_0 + \Im)x^n > (a_k + \Im) \frac{x^n - 1}{x - 1}$$

$$\text{if } x^n > \frac{(a_k + \Im)}{(a_0 + \Im)} (x^n - 1)$$

$$\text{This holds if } \frac{a_k + \Im}{(a_0 + \Im)(x-1)} \leq 1$$

$$\text{If } x \geq \frac{(a_k + \Im)}{(a_0 + \Im)} + 1$$

Hence the theorem.

### 3.5 Multiple roots

Let  $(\alpha_1 + \Im), (\alpha_2 + \Im), \dots, (\alpha_n + \Im)$  be the roots of the equation  $P_N(x) = 0$  in which first  $r(r < n)$  quantities of them be equal to  $(\alpha_1 + \Im)$ , then we write  $P_N(x) = (x_1 - (\alpha_1 + \Im))^r \cdot \phi(x)$ ,  $\phi(\alpha_1 + \Im) \neq 0$  and  $(\alpha_1 + \Im)$  is a root of the equation  $P_N(x) = 0$  of multiplicity  $r$ .

**Theorem2.** If  $(\alpha_1 + \Im)$  be a root of  $P_N(x) = 0$  of multiplicity  $r$ , then  $(\alpha_1 + \Im)$  is a root of  $P'_N(x)$  of multiplicity  $(r-1)$ , where  $P'_N(x)$  is the first order derivative of  $P_N(x)$ .

**Proof.** It is straight forward.

### 3.6 Newtons method of approximation

Let  $P_N(x) = 0$  be a given Neutrosophic polynomial equation in  $x$  and it has a root nearly equal to  $\alpha_1 + \Im$  suppose it is  $\alpha_1 + \Im + h$ , where  $h$  is very very small.

$$P_N(\alpha_1 + \Im + h) = P_N(\alpha_1 + \Im) + h P'_N(\alpha_1 + \Im) + \frac{h^2}{2!} P''_N(\alpha_1 + \Im) + \dots$$

Since  $\alpha_1 + \Im + h$  is a root then  $P_N(\alpha_1 + \Im + h) = 0$

$$P_N(\alpha_1 + \Im) + h P'_N(\alpha_1 + \Im) + \frac{h^2}{2!} P''_N(\alpha_1 + \Im) + \dots = 0$$

Neglecting the square and higher powers of  $h$ , it becomes

$$P_N(\alpha_1 + \Im) + h P'_N(\alpha_1 + \Im) = 0$$

$$\text{or, } h = \frac{-P_N(\alpha_1 + \Im)}{P'_N(\alpha_1 + \Im)}$$

$$\text{Hence the first approximation of the root will be } \beta = \alpha_1 + \Im - \frac{P_N(\alpha_1 + \Im)}{P'_N(\alpha_1 + \Im)}$$

$$\text{Then its closer approximation will be } \beta - \frac{P_N(\beta + \Im)}{P'_N(\beta + \Im)}$$

Proceeding in this manner we may obtain an approximation which is very close to the root upto any desired degree of accuracy. In this way approximation is usually rapid.

**Example7.** Find the real root of  $(1 - \Im)x^3 + (3 + \Im)x + (4 + \Im) = 0$  correct upto 3 significant places.

**Solution.** It is left for the reader.

### 3.7 Symmetric Neutrosophic Functions of Roots

By symmetric neutrosophic functions we mean those functions which remained unchanged in value when any two of its roots are interchanged. We can find the values of symmetric neutrosophic functions of roots in terms of the coefficients.

**Example 8.** If  $(\alpha + \Im), (\beta + \Im), (\gamma + \Im)$  be the roots of  $x^3 - (p + \Im)x^2 + (r + \Im)x = 0$  then, find the equation whose roots are  $\frac{(\beta + \Im) + (\gamma + \Im)}{(\alpha + \Im)}, \frac{(\gamma + \Im) + (\alpha + \Im)}{(\beta + \Im)}, \frac{(\alpha + \Im) + (\beta + \Im)}{(\gamma + \Im)}$

**Solution.**

$$\text{Let } y = \frac{(\beta + \Im) + (\gamma + \Im)}{(\alpha + \Im)} = \frac{(\beta + \Im) + (\gamma + \Im) + (\alpha + \Im) - (\alpha + \Im)}{(\alpha + \Im)} = \frac{(p + \Im)}{(\alpha + \Im)} - 1$$

$$\Rightarrow y + 1 = \frac{(p + \Im)}{(\alpha + \Im)} \Rightarrow \alpha + \Im = \frac{p + \Im}{y + 1}$$

Replacing  $x$  by  $\frac{p + \Im}{y + 1}$  in the equation then it becomes

$$(r + \Im)y^3 + 3(r + 1)y^2 + \left\{ (3r - p^3) + \Im(4 - 3p^2 + 3p) \right\} y + (r + \Im) = 0$$

Here we note that due to the symmetric nature of the roots we have obtained the required equation so easily.

### 3.8 Relation between roots and coefficients of a Neutrosophic polynomial equation

Let  $P_N(x) = (a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + \dots + (a_{n-1} + \Im)x + (a_n + \Im)$  be a Neutrosophic polynomial of degree  $n$ . Let  $P_N(x) = 0$  have  $n$  roots  $(\alpha_1 + \Im), (\alpha_2 + \Im), \dots, (\alpha_n + \Im)$ .

$$(a_0 + \Im)x^n + (a_1 + \Im)x^{n-1} + \dots + (a_{n-1} + \Im)x + (a_n + \Im) = (a_0 + \Im) \left[ \frac{(x - (\alpha_1 + \Im))(x - (\alpha_2 + \Im)) \dots}{(x - (\alpha_n + \Im))} \right]$$

$$\Rightarrow x^n + \frac{(a_1 + \Im)}{(a_0 + \Im)}x^{n-1} + \dots + \frac{(a_{n-1} + \Im)}{(a_0 + \Im)}x + \frac{(a_n + \Im)}{(a_0 + \Im)} = \left[ (x - (\alpha_1 + \Im))(x - (\alpha_2 + \Im)) \dots (x - (\alpha_n + \Im)) \right]$$

$$\Rightarrow x^n + \frac{(a_1 + \Im)}{(a_0 + \Im)}x^{n-1} + \dots + \frac{(a_{n-1} + \Im)}{(a_0 + \Im)}x + \frac{(a_n + \Im)}{(a_0 + \Im)} = x^n - \Sigma(\alpha_1 + \Im)x^{n-1} + \Sigma(\alpha_1 + \Im)(\alpha_2 + \Im)x^{n-2} - \Sigma(\alpha_1 + \Im)(\alpha_2 + \Im)(\alpha_3 + \Im)x^{n-3} + \dots + (-1)^n(\alpha_1 + \Im)(\alpha_2 + \Im) \dots (\alpha_n + \Im)$$

Equating both sides we write the following

$$\Sigma(\alpha_1 + \Im) = -\frac{(a_1 + \Im)}{(a_0 + \Im)}$$

$$\Sigma(\alpha_1 + \Im)(\alpha_2 + \Im) = \frac{(a_2 + \Im)}{(a_0 + \Im)}$$

.....

$$(\alpha_1 + \Im)(\alpha_2 + \Im) \dots (\alpha_n + \Im) = (-1)^n \frac{(a_n + \Im)}{(a_0 + \Im)}$$

#### 4. Conclusions

In this article we have studied neutrosophic real polynomial whose coefficients are neutrosophic numbers. Then we study some of its properties and introduce synthetic method of division to find the quotient and remainder easily. We also give a solution method to find the approximate roots of a neutrosophic polynomial equation. Some concrete examples are given. There is some possibility to use this concept to find the real roots by using different approximation method. Under neutrosophic environment we will use this concept to solve real life problems based on mensuration, trigonometry, geometry etc. We also include some papers in reference section for practical purpose.

**Funding:** “This research received no external funding”

**Conflicts of Interest:** “The authors declare no conflict of interest.”

**Acknowledgement:** The authors thank the editors of the journal and the reviewer for their advice.

#### References

- [1] L. A. Zadeh, “Fuzzy sets”, Inform. Control, Vol 8, pp.338-353, 1965.
- [2] W. L. Gau, and D. J. Buehrer, “Vague sets”, IEEE Transactions on Systems, Man and Cybernetics, Vol 23, pp. 610-614, 1993.

- [3] J. Goguen, “L-fuzzy sets”, *Journal of Mathematical Analysis and Applications*, Vol 18, pp. 145-174, 1967.
- [4] Z. Pawlak, “Rough sets”, *International Journal of Computing and Information Sciences*, Vol 11, pp. 341-356, 1982.
- [5] K. Atanassov, “Intuitionistic fuzzy sets”, *Fuzzy Sets and Systems*, Vol 20, pp. 87-96, 1986.
- [6] M. Gorzalczyk, “A method of inference in approximate reasoning based on interval-valued fuzzy sets”, *Fuzzy Sets and Systems*, Vol 21, pp. 1-17, 1987.
- [7] K. Atanassov and G. Gargov, “Interval-valued intuitionistic fuzzy sets”, *Fuzzy Sets and Systems*, Vol 31, pp. 343-349, 1989.
- [8] F. Smarandache, “Neutrosophic set-a generalization of the intuitionistic fuzzy sets”, *International Journal of Pure and Applied Mathematics*, Vol 24, pp. 287-297, 2005.
- [9] T. Mitra Basu and S. K. Mondal, “Neutrosophic soft matrix and its application in solving group decision making problems from medical science”, *Computer Communication and Collaboration*, Vol 3, pp. 1-31, 2015.
- [10] K. Mondal, S. Pramanik and B.C. Giri, “Hybrid binary logarithm similarity measure for MAGDM problems under SVN assessments”, *Neutrosophic Sets and Systems*, Vol 20, pp. 12-25, 2018.
- [11] K. Mondal, S. Pramanik and B.C. Giri, “Single valued neutrosophic hyperbolic sine similarity measure based MADM strategy”, *Neutrosophic Sets and Systems*, Vol 20, pp. 3-11, 2018.
- [12] M. Sahin, S. Alkhalaleh and V. Ulucay, “Neutrosophic soft expert sets”, *Applied Mathematics*, Vol 6, pp. 116-127, 2015.
- [13] F. Smarandache, “Neutrosophy, neutrosophic probability, set and logic”, *Amer. Res. Press, Rehoboth, USA*, ISBN 978-1879585638, pp. 1-105, 1998.
- [14] D. Molodtsov, “Soft set theory-first results” *Computers and Mathematics with Application*, Vol 37, pp. 19-31, 1999.
- [15] P. K. Maji, R. Biswas and A. R. Roy, “Soft set theory”, *Computers and Mathematics with Applications*, Vol 45, pp. 555-562, 2003.
- [16] P. K. Maji, “Neutrosophic soft set”, *Annals of Fuzzy Mathematics and Informatics*, Vol 5, pp. 157-168, 2013.
- [17] P. K. Maji, “A neutrosophic soft set approach to a decision making problem”, *Annals of Fuzzy mathematics and Informatics*, Vol 3, pp. 313-319, 2012.
- [18] A.A.A. Agboola, “On Refined Neutrosophic Algebraic Structures”, *Neutrosophic Sets and Systems*, Vol 10, pp. 99-101, 2015.
- [19] E.O. Adeleke, A.A.A. Agboola and F. Smarandache, “Refined Neutrosophic Rings I”, *International Journal of Neutrosophic Sciences*, Vol 2, pp. 77-81, 2020.
- [20] E.O. Adeleke, A.A.A. Agboola and F. Smarandache, “Refined Neutrosophic Rings II”, *International Journal of Neutrosophic Sciences*, Vol 2, pp. 89-94, 2020.
- [21] A.A. Salma, S.A. Alblawi, “Neutrosophic set and neutrosophic topological spaces”, *IOSR J. Math*, Vol 3, pp. 31-35, 2012.



## The Polar form of a Neutrosophic Complex Number

Riad K. Al-Hamido<sup>1</sup>, Mayas Ismail<sup>2\*</sup>, Florentin Smarandache<sup>3</sup>

<sup>1</sup> Department of Mathematics, College of Science, AlFurat University, Deir-ez-Zor, Syria.

<sup>1</sup>E-mail: riad-hamido1983@hotmail.com

<sup>2\*</sup> Department of Computer Engineering, International University for Science and Technology, Daraa, Syria.

<sup>2\*</sup>E-mail: mayas.n.ismail@gmail.com; Tel.: (+963937082244)

<sup>3</sup> Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA.

<sup>3</sup>E-mail: smarand@unm.edu

### Abstract

In this paper, we will define the exponential form of a neutrosophic complex number. We have proven some characteristics and theories, including the conjugate of the exponential form of a neutrosophic complex number, division of the exponential form of a neutrosophic complex numbers, multiplication of the exponential form of a neutrosophic complex numbers. In addition, we have given the method of changing from the exponential to the algebraic form of a complex number.

**Keywords:** Neutrosophic numbers, neutrosophic complex number, the exponential form of a neutrosophic complex number.

### 1. Introduction

The American scientist and philosopher F. Smarandache came to place the neutrosophic logic in [1-5], and this logic is as a generalization of the fuzzy logic [6], conceived by L. Zadeh in 1965.

The neutrosophic logic is of great importance in many areas of them, including applications in image processing [7-8], the field of geographic information systems [9], and possible applications to database [10-11], and have applications in the medical field [12-15], and in neutrosophic bitopology in [16-18], and in neutrosophic algebra in [19-23], professor F. Smarandache presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number [24], and Y. Alhasan presented the properties of the concept of neutrosophic complex numbers including the conjugate of neutrosophic complex number, division of neutrosophic complex numbers, the inverted neutrosophic complex number and the absolute value of a neutrosophic complex number and theories related to the conjugate of neutrosophic complex numbers, and that the product of a neutrosophic complex number by its conjugate equals the absolute value of number [25].

This paper aims to study and define the exponential form of a neutrosophic complex number by defining the conjugate of the exponential form of a neutrosophic complex number, division of the exponential form of the neutrosophic complex numbers, and multiplication of the exponential form of a neutrosophic complex numbers.

## 2. Preliminaries

In this section, we present the basic definitions that are useful in this research.

### Definition 2.1 [24]

A neutrosophic number has the standard form:

$$a + bI$$

where  $a, b$  are real or complex coefficients, and  $I =$  indeterminacy, such  $0.I = 0$

$I^n = I$  for all positive integer  $n$ .

If the coefficients  $a$  and  $b$  are real, and then  $a + bI$  is called neutrosophic real number.

For example:  $5+7I$

### Definition 2.2 [25]

$z$  is a neutrosophic complex number, if it takes the following standard form:

$$z = a + bI + ci + diI$$

Where  $a, b, c, d$  are real coefficients, and  $I =$  indeterminacy, and  $i^2 = -1$ .

### Division of Neutrosophic Real Numbers [24]

$$(a_1 + b_1I) \div (a_2 + b_2I) = ?$$

We denote the result by:

$$\frac{a_1 + b_1I}{a_2 + b_2I} = x + yI$$

$$x = \frac{a_1}{a_2}$$

and

$$y = \frac{a_2b_1 - a_1b_2}{a_2(a_2 + b_2)}$$

### Definition 2.4 [25]

Suppose that  $z = a + bI + ci + diI$  is a neutrosophic complex number, then the absolute value of a neutrosophic complex number defined by the following form:

$$|z| = \sqrt{(a + bI)^2 + (c + dI)^2}$$

### 3. The Polar form of a Neutrosophic Complex Number

In this section, we present and study the exponential form of a neutrosophic complex number.

#### Definition 3.1

We define the Exponential Form of a Neutrosophic Complex Number as follows:

$$z = re^{i(\theta+I)}$$

where  $r$  the Absolute Value of the neutrosophic complex number.

#### Remark 3.1.1:

From the general form:

$$z = a + bI + ci + diI$$

$$z = r \left( \frac{a + bI}{r} + \frac{ci + diI}{r} \right)$$

$$z = r \left( \frac{a + bI}{r} + i \cdot \frac{c + dI}{r} \right)$$

#### Remark 3.1.2:

$$r = |z| = \sqrt{(a + bI)^2 + (c + dI)^2}$$

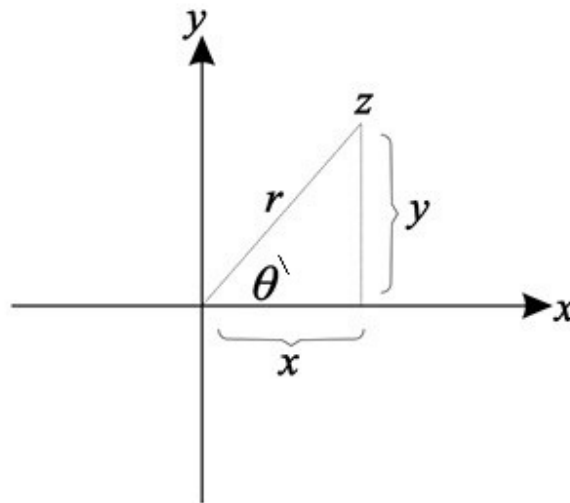


Figure 1: The Geometrical Figure

The formula neutrosophically works in the following way:

$x = a + bI$  is a neutrosophic number whose determinate part is "a" and indeterminate part is "bI", where  $I =$  indeterminacy;

similarly  $y = c + dI$  is a neutrosophic number whose determinate part is "c" and indeterminate part is "dI";

$\Theta = \theta + I$  is a neutrosophic angle, whose determinate part is  $\Theta$  ("theta") and indeterminate part is "I".

It is a big "Theta"  $\Theta$  (inside the geometrical figure) and small "theta"  $\theta$  in the formulas.

That means that we work with two lengths  $x$  and  $y$  that are not well-known (they were approximated), and an angle  $\Theta$  (Theta) that is not well known either (it was approximated by  $\theta$  plus some indeterminacy  $I$ ).

$$\cos(\theta + I) = \frac{x}{r} = \frac{a + bI}{r}, \quad \sin(\theta + I) = \frac{y}{r} = \frac{c + dI}{r}$$

$$z = r(\cos(\theta + I) + i \cdot \sin(\theta + I))$$

Exponential Form:

$$z = re^{i(\theta + I)}$$

### Definition 3.2

Trigonometric formula

$$z = r(\cos(\theta + I) + i \sin(\theta + I))$$

#### 4. Properties

In this section, we present some important properties of the exponential form.

##### Multiplying the exponential forms of the neutrosophic complex numbers

Suppose that  $z_1, z_2$  are two neutrosophic complex numbers, where

$$z_1 = r_1 e^{i(\theta_1 + I_1)} \text{ and } z_2 = r_2 e^{i(\theta_2 + I_2)}$$

If  $I_1 + I_2 = I$

##### Definition 4.1

$$z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2 + I)}$$

##### Remark 4.1.1:

$$z_1 \cdot z_2 = r_1 e^{i(\theta_1 + I_1)} \cdot r_2 e^{i(\theta_2 + I_2)}$$

$$z_1 \cdot z_2 = r_1 r_2 (e^{i(\theta_1 + I_1)} \cdot e^{i(\theta_2 + I_2)})$$

$$z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2 + I_1 + I_2)}$$

$$I_1 + I_2 = I$$

Then

$$z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2 + I)}$$

##### Example 4.1.2

If  $z_1 = r_1 e^{i(\frac{\pi}{4} + I)}$  and  $z_2 = r_2 e^{i(\frac{3\pi}{4} + I)}$

$$z_1 \cdot z_2 = r_1 r_2 e^{i(\frac{\pi}{4} + \frac{3\pi}{4} + I)} = r_1 r_2 e^{i(\pi + I)}$$

##### Division of the exponential forms of neutrosophic complex numbers

Suppose that  $z_1, z_2$  are two neutrosophic complex numbers, where

$$z_1 = r_1 e^{i(\theta_1 + I_1)} \text{ and } z_2 = r_2 e^{i(\theta_2 + I_2)}$$

If  $I_1 - I_2 = I$

then

##### Definition 4.2

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2 + I)}$$

**Remark 4.2.1:**

Depending on [25]

$$z \cdot \bar{z} = |z|^2 = r^2$$

When  $r=1$  we get  $\Rightarrow$

$$\bar{z} = \frac{1}{z} = \frac{e^0}{e^{i(\theta+I)}} = e^{-i(\theta+I)}$$

Then

$$\frac{z_1}{z_2} = \frac{r_1 e^{i(\theta_1 + I_1)}}{r_2 e^{i(\theta_2 + I_2)}}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \left( \frac{e^{i(\theta_1 + I_1)}}{e^{i(\theta_2 + I_2)}} \right)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \left( e^{i(\theta_1 + I_1)} \cdot \frac{1}{e^{i(\theta_2 + I_2)}} \right)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \left( e^{i(\theta_1 + I_1)} \cdot e^{-i(\theta_2 + I_2)} \right)$$

$$I_1 - I_2 = I$$

Then

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2 + I)}$$

**Example 4.2.2**

If  $z_1 = r_1 e^{i(\frac{\pi}{4} + I)}$  and  $z_2 = r_2 e^{i(\frac{3\pi}{4} + I)}$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\frac{\pi}{4} - \frac{3\pi}{4} + I)} = \frac{r_1}{r_2} e^{-i(\frac{\pi}{2} + I)}$$

**The conjugate of the exponential form of a neutrosophic complex numbers 4.3**

Suppose that  $z$  is a neutrosophic complex number, where

$$z = r e^{i(\theta+I)}$$

We denote the conjugate of a neutrosophic complex number by  $\bar{z}$  and define it by the following form:

$$\bar{z} = r e^{-i(\theta+I)}$$

#### Example 4.3.1

$$z = r e^{i(\frac{\pi}{2}+I)}$$

$$\bar{z} = r e^{-i(\frac{\pi}{2}+I)}$$

#### Remark 4.4

If  $I=0$  we will return to the basic formula for the complex number.

$$z = r e^{i(\theta+0)}$$

$$z = r e^{i(\theta)}$$

#### Conclusion

In this paper, we defined the exponential form of a neutrosophic complex number and demonstrated this with appropriate proof, and many examples were presented to illustrate the concepts introduced in this paper.

#### Future Research Directions

As a future work, some special cases related to exponential form can be discussed and benefit from this article in many engineering sciences, including theories of control and signal processing.

#### References

- [1] F. Smarandache, "Neutrosophic Precalculus and Neutrosophic Calculus", EuropaNova asbl, Clos du Parnasse, 3E 1000, Bruxelles, Belgium, 2015.
- [2] F. Smarandache, "An Introduction to Neutrosophic Probability Applied in Quantum Physics", Bulletin of the American Physical Society, 2009 APS April Meeting, Volume 54, Number 4, Saturday–Tuesday, May 2–5, 2009; Denver, Colorado, USA, <http://meetings.aps.org/Meeting/APR09/Event/102001>.
- [3] F. Smarandache, "An Introduction to Neutrosophic Probability Applied in Quantum Physics", SAO/NASA ADS Physics Abstract Service, <http://adsabs.harvard.edu/abs/2009APS..APR.E1078S>.

- [4] F. Smarandache, "A Unifying Field in Logics: Neutrosophic Logic, Neutrosophic Set", Neutrosophic Probability and Statistics, (1999), (2000), (2003), (2005).
- [5] F. Smarandache; Huda E.Khalid,"Neutrosophic Precalculus and Neutrosophic Calculus", Second enlarged edition, Pons asbl, 5, Quai du Batelage, Brussels, Belgium, European Union, 2018.
- [6] L. Zadeh, "Fuzzy sets, Inform and Control", 8, pp.338-353, 1965.
- [7] A. A Salama; I. M Hanafy; Hewayda Elghawalby Dabash M.S,"Neutrosophic Crisp Closed Region and Neutrosophic Crisp Continuous Functions", New Trends in Neutrosophic Theory and Applications.
- [8] A. A Salama; Hewayda Elghawalby; M.S. Dabash; A. M. NASR, "Retrac Neutrosophic Crisp System For Gray Scale Image", Asian Journal of Mathematics and Computer Research, Vol. 24, 104-117-22, 2018.
- [9] A. A Salama,"Basic Structure of Some Classes of Neutrosophic Crisp Nearly Open Sets & Possible Application to GIS Topology", Neutrosophic Sets and Systems, Vol. 7, 18-22, 2015.
- [10] A. A Salama, F. Smarandache," Neutrosophic Crisp Set Theory", Neutrosophic Sets and Systems, Vol. 5, 1-9, 2014.
- [11] F. Smarandache, "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics", University of New Mexico, Gallup, NM 87301, USA ,2002.
- [12] M. Abdel-Basset; M. Mai, E. Mohamed; C. Francisco;H. Z. Abd El-Nasser, "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases", Artificial Intelligence in Medicine Vol. 101 , 101735,2019.
- [13] M. Abdel-Basset; E. Mohamed; G. Abdullallah; and S. Florentin, "A novel model for evaluation Hospital medical care systems based on plithogenic sets", Artificial intelligence in medicine 100, 101710, 2019.
- [14] M. Abdel-Basset; G. Gunasekaran Mohamed; G. Abdullallah. C. Victor,"A Novel Intelligent Medical Decision Support Model Based on Soft Computing and IoT", IEEE Internet of Things Journal, Vol. 7, 2019.
- [15] M. Abdel-Basset; G. Abdullallah; G. Gunasekaran; L. Hoang Viet, "A novel group decision making model based on neutrosophic sets for heart disease diagnosis", Multimedia Tools and Applications, pp.1-26, 2019.
- [16] R.K. Al-Hamido,"Neutrosophic Crisp Supra Bi-Topological Spaces", International Journal of Neutrosophic Science, Vol. 1, 66-73, 2018.
- [17] R.K. Al-Hamido,"Neutrosophic Crisp Bi-Topological Spaces", Neutrosophic Sets and Systems, vol. 21, pp.66-73, 2018.
- [18] R.K. Al-Hamido; T. Gharibah; S. Jafari, F.Smarandache, "On Neutrosophic Crisp Topology via N-Topology", Neutrosophic Sets and Systems, vol. 21, 96-109, 2018.
- [19] V. Christianto; F. Smarandache; M. Aslam, "How we can extend the standard deviation notion with neutrosophic interval and quadruple neutrosophic numbers", International Journal of Neutrosophic Science, vol. 2, No. 2, pp.72-76 , 2020.
- [20] E. O. Adeleke; A. A. A. Agboola; F. Smarandache, "Refined Neutrosophic Rings I", International Journal of Neutrosophic Science, vol. 2 , No. 2, pp.77-81 , 2020.
- [21] E. O. Adeleke; A. A. A. Agboola; F. Smarandache, "Refined Neutrosophic Rings II", International Journal of Neutrosophic Science, vol 2. , No. 2, pp.89-94 , 2020.
- [22] M. A. Ibrahim; A. A. A. Agboola; E. O. Adeleke; S. A. Akinleye, "Introduction to Neutrosophic Subtraction Algebra and Neutrosophic Subtraction Semigroup", International Journal of Neutrosophic Science, vol. 2 , No. 1, pp.47-62 , 2020.
- [23] A. Hatip,"The Special Neutrosophic Functions", International Journal of Neutrosophic Science, vol. 4 , No. 2, pp.104-116, 2020.
- [24] F. Smarandache, "Introduction to Neutrosophic Statistics", Sitech-Education Publisher, pp.34-44, 2014.
- [25] Y. Alhasan, "Concepts of Neutrosophic Complex Numbers ", International Journal of Neutrosophic Science, Vol. 8 No. 1, pp. 9-18, 2020.
- [26] Samah Ibrahim Abdel Aal, Mahmoud M. A. Abd Ellatif, Mohamed Monir Hassan: Two Ranking Methods of Single Valued Triangular Neutrosophic Numbers to Rank and Evaluate Information Systems Quality, Neutrosophic Sets and Systems, vol. 19,pp. 132-141, 2018. <http://doi.org/10.5281/zenodo.1235339>

- [27] Faruk Karaaslan: Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making, *Neutrosophic Sets and Systems*, vol. 22, pp. 101-117, 2018. DOI: 10.5281/zenodo.2159762
- [28] Surapati Pramanik, Rama Mallick: VIKOR based MAGDM Strategy with Trapezoidal Neutrosophic Numbers, *Neutrosophic Sets and Systems*, vol. 22, pp. 118-130, 2018. DOI: 10.5281/zenodo.2160840
- [29] Irfan Deli: Operators on Single Valued Trapezoidal Neutrosophic Numbers and SVTN-Group Decision Making, *Neutrosophic Sets and Systems*, vol. 22, pp. 131-150, 2018. DOI: 10.5281/zenodo.2160120
- [30] Bibhas C. Giri, Mahatab Uddin Molla, Pranab Biswas: TOPSIS Method for MADM based on Interval Trapezoidal Neutrosophic Number, *Neutrosophic Sets and Systems*, vol. 22, pp. 151-167, 2018. DOI: 10.5281/zenodo.2160749



## Nonagonal Neutrosophic Linear Non-Linear Numbers, Alpha Cuts and Their Applications using TOPSIS

Muhammad Naveed Jafar<sup>1</sup>, Ezgi TÜRKARSLAN<sup>2</sup>, Ali Hamza<sup>3</sup>, Sara Farooq<sup>3</sup>

<sup>1</sup>Department of Mathematics Lahore Garrison University, Av-4, Sec-C, DHA Phase VI, 54000 Lahore Pakistan

<sup>2</sup>Department of Mathematics, TED University, Ön Cebeci, Ziya Gökalp Cd. 48/A, 06420 Çankaya/Ankara, Turkey

Email: [ezgi.turkarslan@tedu.edu.tr](mailto:ezgi.turkarslan@tedu.edu.tr)

<sup>3</sup>Department of Mathematics Lahore Garrison University, Av-4, Sec-C, DHA Phase VI, 54000 Lahore Pakistan

<sup>3</sup>Department of Mathematics Lahore Garrison University, Av-4, Sec-C, DHA Phase VI, 54000 Lahore Pakistan

Correspondence: [naveedjafar@lgu.edu.pk](mailto:naveedjafar@lgu.edu.pk)

### Abstract

The concept of neutrosophic become really handy now a days and based on non-standard analysis to mention mathematical outcomes, uncertainty, non-completed situations, inconsistency, distinctness. The main concept of Neutrosophic set based on membership values of truth, indeterminacy and falsity, which are independent and which play vital role in situations like uncertainty, incomplete and inconsistency. From triangular to octagonal neutrosophic number. They play vital role in modeling problems, science, biology and many more. Hence it is clear that these are necessary and have real life applications, but some real-life problems have more edges and their triangular to octagonal fail to overcome this situation (mention in table 1). Hence, nonagonal neutrosophic numbers give a wide scope of utilizations while managing more variances in the decision-making condition with nine edges for membership values of truth, indeterminacy and falsity. In this current article we present comparison between triangular to nonagonal neutrosophic number and their requirement, explore differential equations in neutrosophic environment as Linear, symmetric and asymmetric types further, their  $\alpha$  – cut and then we present a real-life problem and solved it with TOPSIS technique of MCDM.

**Keywords:** Accuracy function, Neutrosophic number, Nonagonal Neutrosophic numbers (NNN), MCDM, TOPSIS.

### 1- Introduction

Analysts and mathematicians from throughout the world created significant expository aptitudes and critical thinking systems to survey an expansive scope of issues in human asset, medication, determination issues and so on but the most testing issues was related with MCDM.

In this way, the need to deal with unsure circumstances and dubiousness, mathematicians generate a new way, called fuzzy which can fulfil their decision-making requirements. The very first attempt in this race was soft set theory which was introduced by Molodtsov [1] in 1999 to deal with uncertainty. The further effort was fuzzy set theory [2-4] then intuitionistic fuzzy numbers [5] and finally neutrosophic sets [6] as figure 2. Neutrosophic fuzzy numbers [7] is really

quintessential regards to decision making and triangular neutrosophic numbers [8] was commencing then Trapezoidal neutrosophic numbers then Pentagonal neutrosophic numbers [9] then Hexagonal neutrosophic numbers then heptagonal neutrosophic numbers then octagonal neutrosophic numbers [10], his representations and  $\tilde{\alpha}$  – cut was given as [11] as well as his aggregate, arithmetic and geometric operators of octagonal neutrosophic numbers and applications also proposed [12] and finally we proposed nonagonal neutrosophic numbers, as well as researchers keep working and introduce new possibilities [13-15]. Wang [16] extended the idea and present single-value neutrosophic sets as extension of NSs. Simplified neutrosophic sets given by Ye [17] and novel operations and aggregation operation was given by Peng et al [16,17]. Multi-valued [18], bipolar [19] and interval neutrosophic sets [20] are different extensions of neutrosophic sets.

Smarandache and different researchers [21-28] from all over the world work on various extension of neutrosophic sets in MCDM and TOPSIS technique. A lot work available regards to selection problem [29-49] and MCDM. Properties as well as applications of triangular and pentagonal neutrosophic number given by [50-54]. With these unique concepts of Nonagonal Neutrosophic numbers (NNN), we can work on many fluctuations at the same time, that proceedable due to more edges found in nonagonal as compare to triangular, pentagonal and octagonal. Now with this current epoch, we can convert neutrosophic numbers into fuzzy numbers and also the power to deal with more fluctuations move on a new level. With these great efforts of mathematicians, we are able to perform these productive and value able decision-making efforts.

Neutrosophic numbers also really useful in other mathematical fields such as transportation problem in neutrosophic environment of operational research [55]. Then, Pratihari further introduce modified Vogel's approximation method [56] in the field of operational research, a method to increase income. The concept of single inputs and outputs of pipelines further improve by Mohapatra [57]. In the line of progress of operational research Kumar introduce optimal path selection by fuzzy reliable shortest path [58] and neutrosophic shortest path [59]. Gayen introduce anti-fuzzy subgroup [60] and plithogenic subgroup [61-62], which further promote the plithogenic algebraic structure as well as introduce notation of plithogenic subgroup. Gayen with smarandache introduce interval-valued triple T-norm [63], which have both pure and applied math applications.

In the case of nonagonal neutrosophic number, we have nine edges regards to truthness, Indeterminacy and Falsity membership value. So, provide us more range to deal with more fluctuations. As decision making is really diverse and come with endless possibilities, hence we required a system which can be useable in these tuff situations. Detail example available as table below:

Neutrosophic numbers	Black color	High mileage	Price	Design	Speed	Safety Features	4×4	Resale value	Break system
<b>Triangular Neutrosophic numbers</b>	Determinable (D)	(D)	(D)	*	*	*	*	*	*
<b>Trapezoidal Neutrosophic numbers</b>	(D)	(D)	(D)	(D)	*	*	*	*	*
<b>Pentagonal Neutrosophic numbers</b>	(D)	(D)	(D)	(D)	(D)	*	*	*	*
<b>Hexagonal Neutrosophic numbers</b>	(D)	(D)	(D)	(D)	(D)	(D)	*	*	*
<b>Heptagonal NNs</b>	(D)	(D)	(D)	(D)	(D)	(D)	(D)	*	*
<b>Octagonal Neutrosophic numbers</b>	(D)	(D)	(D)	(D)	(D)	(D)	(D)	(D)	*

<b>Nonagonal Neutrosophic numbers</b>	(D)	(D)	(D)	(D)	(D)	(D)	(D)	(D)	(D)
---------------------------------------	-----	-----	-----	-----	-----	-----	-----	-----	-----

**Table 1:** This current table shows, if we have to take a decision on nine different criteria then only nonagonal neutrosophic numbers can do that job for us.

### 1.1 Motivation

A lot of research articles regards to neutrosophic arena available, which they apply and exaggerated the concept of MCDM. Here we have a ground braking concept, nonagonal neutrosophic numbers (NNN), which is totally new. Our main approach is to define the concept of nonagonal for that cenerio, where triangular to octangonal neutrosophic fail to overcome. With the nine edges and wide range of membership values of truth, indeterminacy and falsity, it can proceed better and overcome more real life problems, based on uncertainty.

### 1.2 The Paper presentation

We extended the concept of Nonagonal Neutrosophic Numbers (NNN).

- Introduced Linear, Non-Linear, Linear symmetric, Non-Linear symmetric Nonagonal neutrosophic Numbers.
- $\tilde{\alpha}$  – cut is defined for all type.
- A case study which is defined from real-life selection problem, solved by TOPSIS.

### 1.3 Structure of Paper

The structure of this current epoch is defined in following figure.

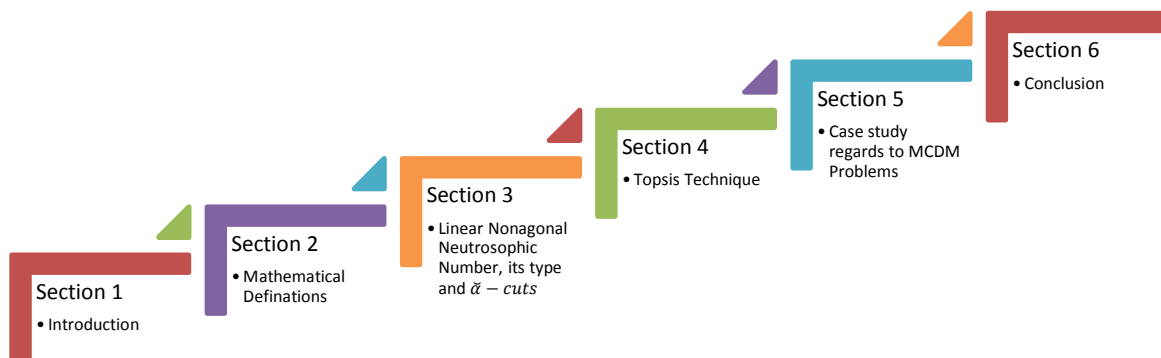


Figure 1: Structure of article with a pictorial view

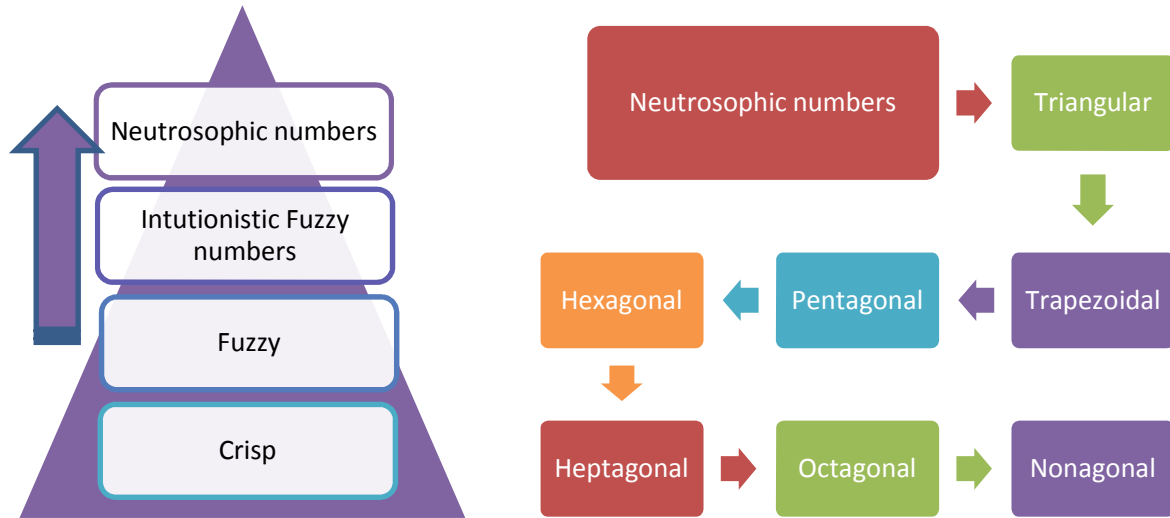


Figure 2. Extensions of intuitionistic fuzzy numbers as flow chart.

## 2 Mathematical Definitions

In that section, we will mention necessary required definitions, which we will use throughout the paper.

**Definition 2.1: Neutrosophic Sets:** Set  $\widetilde{nA}$  is neutrosophic if  $\widetilde{nA} = \{x; \langle [T_{\widetilde{nA}}(x), I_{\widetilde{nA}}(x), F_{\widetilde{nA}}(x)] \rangle : x \in X\}$

Where  $T_{\widetilde{nA}}(x) \mapsto [0, 1]$  as truth membership function (TMF), indeterminacy IMF  $I_{\widetilde{nA}}(x)$ , and falsity (FMF)  $F_{\widetilde{nA}}(x)$  as well as  $T_{\widetilde{nA}}(x), I_{\widetilde{nA}}(x), F_{\widetilde{nA}}(x)$  mentions following relation:

$$0^- \leq T_{\widetilde{nA}}(x) + I_{\widetilde{nA}}(x) + F_{\widetilde{nA}}(x) \leq 3^+$$

**Definition 2.2: Triangular Neutrosophic numbers:** Triangular single value neutrosophic number is given as  $A_{\widetilde{neu}} = (\check{p}_1, \check{p}_2, \check{p}_3; \check{r}_1, \check{r}_2, \check{r}_3)$  as well as truth, indeterminacy and falsity is given as:

$$\begin{aligned} \check{T}_{A_{\widetilde{neu}}} &= \begin{cases} \frac{x - \check{p}_1}{\check{p}_1 - \check{p}_2} & \text{for } \check{p}_1 \leq x < \check{p}_2 \\ 1 & \text{when } x = \check{p}_2 \\ \frac{\check{p}_3 - x}{\check{p}_3 - \check{p}_2} & \text{for } \check{p}_2 < x \leq \check{p}_3 \\ 0 & \text{otherwise} \end{cases} \\ \check{I}_{A_{\widetilde{neu}}} &= \begin{cases} \frac{\check{q}_2 - x}{\check{q}_2 - \check{q}_1} & \text{for } \check{q}_1 \leq x < \check{q}_2 \\ 0 & \text{when } x = \check{q}_2 \\ \frac{x - \check{q}_2}{\check{q}_3 - \check{q}_2} & \text{for } \check{q}_2 < x \leq \check{q}_3 \\ 1 & \text{otherwise} \end{cases} \\ \check{F}_{A_{\widetilde{neu}}} &= \begin{cases} \frac{x - \check{p}_1}{\check{p}_1 - \check{p}_2} & \text{for } \check{p}_1 \leq x < \check{p}_2 \\ 1 & \text{when } x = \check{p}_2 \\ \frac{\check{p}_3 - x}{\check{p}_3 - \check{p}_2} & \text{for } \check{p}_2 < x \leq \check{p}_3 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{Where } 0 \leq T_{A_{\text{Neu}}}(x) + I_{A_{\text{Neu}}}(x) + F_{A_{\text{Neu}}}(x) \leq 3, x \in A_{\text{Neu}}$$

Parametric foam for this type is  $(A_{\text{Neu}})_{\alpha, \beta, \gamma} = [\tilde{T}_{\text{Neu}1}(\tilde{\alpha}), \tilde{T}_{\text{Neu}2}(\tilde{\alpha}); \tilde{I}_{\text{Neu}1}(\tilde{\alpha}), \tilde{I}_{\text{Neu}2}(\tilde{\alpha}); \tilde{F}_{\text{Neu}1}(\tilde{\alpha}), \tilde{F}_{\text{Neu}2}(\tilde{\alpha})]$ , where,  $\tilde{T}_{\text{Neu}1}(\tilde{\alpha}) = P_1 + \tilde{\alpha}(\ddot{P}_2 - P_1)$ ,  $\tilde{T}_{\text{Neu}2}(\tilde{\alpha}) = P_3 - \tilde{\alpha}(\ddot{P}_3 - P_2)$ ,  $\tilde{I}_{\text{Neu}1}(\tilde{\beta}) = q_2 - \tilde{\beta}(\ddot{q}_2 - q_1)$ ,  $\tilde{I}_{\text{Neu}2}(\tilde{\beta}) = q_2 + \tilde{\beta}(\ddot{q}_3 - q_2)$ ,  $\tilde{F}_{\text{Neu}1}(\tilde{\gamma}) = r_2 - \tilde{\gamma}(\ddot{r}_2 - r_1)$ ,  $\tilde{F}_{\text{Neu}2}(\tilde{\gamma}) = r_2 + \tilde{\gamma}(\ddot{r}_3 - r_2)$ , here  $0 < \tilde{\alpha} \leq 1, 0 < \tilde{\beta} \leq 1, 0 < \tilde{\gamma} \leq 1$  and  $0 < \tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} < 3$ .

**Definition 2.3: Trapezoidal Neutrosophic Number:** Let  $\tilde{X}$  as universe if discourse, A trapezoidal neutrosophic set  $\tilde{A}$  in  $\tilde{X}$  is defined as:  $\tilde{N} = \{(\tilde{X}, \tilde{T}_{\tilde{N}}(x), \tilde{I}_{\tilde{N}}(x), \tilde{F}_{\tilde{N}}(x)) | x \in \tilde{X}\}$  where  $\tilde{T}_{\tilde{N}}(x) \subset [0, 1]$ ,  $\tilde{I}_{\tilde{N}}(x) \subset [0, 1]$ ,  $\tilde{F}_{\tilde{N}}(x) \subset [0, 1]$  as three trapezoidal number,  $\tilde{T}_{\tilde{N}}(x) = (\tilde{t}_N^1(x), \tilde{t}_N^2(x), \tilde{t}_N^3(x), \tilde{t}_N^4(x)) : x \mapsto [0, 1]$ ,  $\tilde{I}_{\tilde{N}}(x) = (\tilde{i}_N^1(x), \tilde{i}_N^2(x), \tilde{i}_N^3(x), \tilde{i}_N^4(x)) : x \mapsto [0, 1]$ ,  $\tilde{F}_{\tilde{N}}(x) = (\tilde{f}_N^1(x), \tilde{f}_N^2(x), \tilde{f}_N^3(x), \tilde{f}_N^4(x)) : x \mapsto [0, 1]$  with condition  $0 \leq \tilde{t}_N^4(x) + \tilde{i}_N^4(x) + \tilde{f}_N^4(x) \leq 3, x \in \tilde{X}$ .

**Definition 2.4: Pentagonal Neutrosophic Number:** For single valued, pentagonal neutrosophic number( $\tilde{S}$ ) given as  $\tilde{S} = \langle [\tilde{m}^1, \tilde{n}^1, \tilde{o}^1, \tilde{p}^1, \tilde{q}^1; \tilde{\pi}], [\tilde{m}^2, \tilde{n}^2, \tilde{o}^2, \tilde{p}^2, \tilde{q}^2; \tilde{\xi}], [\tilde{m}^3, \tilde{n}^3, \tilde{o}^3, \tilde{p}^3, \tilde{q}^3; \tilde{\delta}] \rangle$  where  $\tilde{\pi}, \tilde{\xi}, \tilde{\delta} \in [0, 1]$ . Truth membership function ( $\tilde{T}_{\tilde{S}} : \mathbb{R} \mapsto [0, \tilde{\pi}]$ ), the indeterminacy ( $\tilde{I}_{\tilde{S}} : \mathbb{R} \mapsto [0, \tilde{\xi}]$ ) and falsity ( $\tilde{F}_{\tilde{S}} : \mathbb{R} \mapsto [0, \tilde{\delta}]$ ) as well as given as:

$$T_{A_{\text{Neu}}} = \begin{cases} \tilde{T}_{St1}(x) & \tilde{m}^1 \leq x < \tilde{n}^1 \\ \tilde{T}_{St2}(x) & \tilde{n}^1 \leq x < \tilde{o}^1 \\ \tilde{\pi} & x = \tilde{o}^1 \\ \tilde{T}_{St2}(x) & \tilde{o}^1 \leq x < \tilde{p}^1 \\ \tilde{T}_{St1}(x) & \tilde{p}^1 \leq x < \tilde{q}^1 \\ 0 & \text{otherwise} \end{cases} \quad I_{A_{\text{Neu}}} = \begin{cases} \tilde{I}_{St1}(x) & \tilde{m}^2 \leq x < \tilde{n}^2 \\ \tilde{I}_{St2}(x) & \tilde{n}^2 \leq x < \tilde{o}^2 \\ \tilde{\xi} & x = \tilde{o}^2 \\ \tilde{I}_{St2}(x) & \tilde{o}^2 \leq x < \tilde{p}^2 \\ \tilde{I}_{St1}(x) & \tilde{p}^2 \leq x < \tilde{q}^2 \\ 1 & \text{otherwise} \end{cases} \quad F_{A_{\text{Neu}}} = \begin{cases} \tilde{F}_{St1}(x) & \tilde{m}^3 \leq x < \tilde{n}^3 \\ \tilde{F}_{St2}(x) & \tilde{n}^3 \leq x < \tilde{o}^3 \\ \tilde{\delta} & x = \tilde{o}^3 \\ \tilde{F}_{St2}(x) & \tilde{o}^3 \leq x < \tilde{p}^3 \\ \tilde{F}_{St1}(x) & \tilde{p}^3 \leq x < \tilde{q}^3 \\ 1 & \text{otherwise} \end{cases}$$

Where  $\langle [\tilde{m}^1 < \tilde{n}^1 < \tilde{o}^1 < \tilde{p}^1 < \tilde{q}^1; \tilde{\pi}], [\tilde{m}^2 < \tilde{n}^2 < \tilde{o}^2 < \tilde{p}^2 < \tilde{q}^2; \tilde{\xi}], [\tilde{m}^3 < \tilde{n}^3 < \tilde{o}^3 < \tilde{p}^3 < \tilde{q}^3; \tilde{\delta}] \rangle$

**Definition 2.5: Octagonal Neutrosophic Number [ONN]:** Neutrosophic as  $\tilde{S}$  will defined as,

$\tilde{S} = \langle [(\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{e}, \tilde{f}, \tilde{g}, \tilde{h}) : \tilde{\theta}] [(\tilde{a}^1, \tilde{b}^1, \tilde{c}^1, \tilde{d}^1, \tilde{e}^1, \tilde{f}^1, \tilde{g}^1, \tilde{h}^1) : \tilde{\psi}] [(\tilde{a}^2, \tilde{b}^2, \tilde{c}^2, \tilde{d}^2, \tilde{e}^2, \tilde{f}^2, \tilde{g}^2, \tilde{h}^2) : \tilde{\varphi}] \rangle$  where  $\tilde{\theta}, \tilde{\psi}, \tilde{\varphi} \in [0, 1]$ .

The truth membership function ( $\tilde{\theta}_{\tilde{S}} : \mathbb{R} \mapsto [0, 1]$ ),

The indeterminacy membership function ( $\tilde{\psi}_{\tilde{S}} : \mathbb{R} \mapsto [0, 1]$ ),

The falsity membership function ( $\tilde{\varphi}_{\tilde{S}} : \mathbb{R} \mapsto [0, 1]$ ) as well as given follows:

$$\tilde{\theta}_{\tilde{S}}(x) = \begin{cases} \tilde{\theta}_{S0}(x) & \tilde{a} \leq x < \tilde{b} \\ \tilde{\theta}_{S1}(x) & \tilde{b} \leq x < \tilde{c} \\ \tilde{\theta}_{S2}(x) & \tilde{c} \leq x < \tilde{d} \\ \tilde{\theta}_{S3}(x) & \tilde{d} \leq x < \tilde{e} \\ \tilde{\theta} & x = \tilde{e} \\ \tilde{\theta}_{S3}(x) & \tilde{e} \leq x < \tilde{f} \\ \tilde{\theta}_{S2}(x) & \tilde{f} \leq x < \tilde{g} \\ \tilde{\theta}_{S1}(x) & \tilde{g} \leq x < \tilde{h} \\ 0 & \text{otherwise} \end{cases} \quad \tilde{\psi}_{\tilde{S}}(x) = \begin{cases} \tilde{\psi}_{S0}(x) & \tilde{a}^1 \leq x < \tilde{b}^1 \\ \tilde{\psi}_{S1}(x) & \tilde{b}^1 \leq x < \tilde{c}^1 \\ \tilde{\psi}_{S2}(x) & \tilde{c}^1 \leq x < \tilde{d}^1 \\ \tilde{\psi}_{S3}(x) & \tilde{d}^1 \leq x < \tilde{e}^1 \\ \tilde{\psi} & x = \tilde{e}^1 \\ \tilde{\psi}_{S3}(x) & \tilde{e}^1 \leq x < \tilde{f}^1 \\ \tilde{\psi}_{S2}(x) & \tilde{f}^1 \leq x < \tilde{g}^1 \\ \tilde{\psi}_{S1}(x) & \tilde{g}^1 \leq x < \tilde{h}^1 \\ 1 & \text{otherwise} \end{cases} \quad \tilde{\varphi}_{\tilde{S}}(x) = \begin{cases} \tilde{\varphi}_{S0}(x) & \tilde{a}^2 \leq x < \tilde{b}^2 \\ \tilde{\varphi}_{S1}(x) & \tilde{b}^2 \leq x < \tilde{c}^2 \\ \tilde{\varphi}_{S2}(x) & \tilde{c}^2 \leq x < \tilde{d}^2 \\ \tilde{\varphi}_{S3}(x) & \tilde{d}^2 \leq x < \tilde{e}^2 \\ \tilde{\varphi} & x = \tilde{e}^2 \\ \tilde{\varphi}_{S3}(x) & \tilde{e}^2 \leq x < \tilde{f}^2 \\ \tilde{\varphi}_{S2}(x) & \tilde{f}^2 \leq x < \tilde{g}^2 \\ \tilde{\varphi}_{S1}(x) & \tilde{g}^2 \leq x < \tilde{h}^2 \\ 1 & \text{otherwise} \end{cases}$$

Where  $\check{S} = \langle [(\check{a} < \check{b} < \check{c} < \check{d} < \check{e} < \check{f} < \check{g} < \check{h}): \check{\theta}] [(\check{a}^1 < \check{b}^1 < \check{c}^1 < \check{d}^1 < \check{e}^1 < \check{f}^1 < \check{g}^1 < \check{h}^1): \psi] [(\check{a}^2 < \check{b}^2 < \check{c}^2 < \check{d}^2 < \check{e}^2 < \check{f}^2 < \check{g}^2 < \check{h}^2): \varphi] \rangle$ .

**Definition 2.6: Nonagonal Neutrosophic Number [NNN]:** Neutrosophic as  $\check{S}$  will defined as,

$\check{S} = \langle [(\check{a}, \check{b}, \check{c}, \check{d}, \check{e}, \check{f}, \check{g}, \check{h}, \check{i}): \check{\theta}] [(\check{a}^1, \check{b}^1, \check{c}^1, \check{d}^1, \check{e}^1, \check{f}^1, \check{g}^1, \check{h}^1, \check{i}^1): \psi] [(\check{a}^2, \check{b}^2, \check{c}^2, \check{d}^2, \check{e}^2, \check{f}^2, \check{g}^2, \check{h}^2, \check{i}^2): \varphi] \rangle$  where  $\check{\theta}, \check{\psi}, \check{\varphi} \in [0,1]$ .

The truth membership function  $(\check{\theta}_S): \mathbb{R} \mapsto [0,1]$ ,

The indeterminacy membership function  $(\check{\psi}_S): \mathbb{R} \mapsto [0,1]$ ,

The falsity membership function  $(\check{\varphi}_S): \mathbb{R} \mapsto [0,1]$  as well as given follows:

$$\check{\theta}_S(x) = \begin{cases} \check{\theta}_{s0}(x) & \check{a} \leq x < \check{b} \\ \check{\theta}_{s1}(x) & \check{b} \leq x < \check{c} \\ \check{\theta}_{s2}(x) & \check{c} \leq x < \check{d} \\ \check{\theta}_{s3}(x) & \check{d} \leq x < \check{e} \\ \check{\theta} & x = \check{e} \\ \check{\theta}_{s3}(x) & \check{e} \leq x < \check{f} \\ \check{\theta}_{s2}(x) & \check{f} \leq x < \check{g} \\ \check{\theta}_{s1}(x) & \check{g} \leq x < \check{h} \\ \check{\theta}_{s0}(x) & \check{h} \leq x < \check{i} \\ 0 & \text{otherwise} \end{cases} \quad \check{\psi}_S(x) = \begin{cases} \check{\psi}_{s0}(x) & \check{a}^1 \leq x < \check{b}^1 \\ \check{\psi}_{s1}(x) & \check{b}^1 \leq x < \check{c}^1 \\ \check{\psi}_{s2}(x) & \check{c}^1 \leq x < \check{d}^1 \\ \check{\psi}_{s3}(x) & \check{d}^1 \leq x < \check{e}^1 \\ \check{\psi} & x = \check{e}^1 \\ \check{\psi}_{s3}(x) & \check{e}^1 \leq x < \check{f}^1 \\ \check{\psi}_{s2}(x) & \check{f}^1 \leq x < \check{g}^1 \\ \check{\psi}_{s1}(x) & \check{g}^1 \leq x < \check{h}^1 \\ \check{\psi}_{s0}(x) & \check{h}^1 \leq x < \check{i}^1 \\ 1 & \text{otherwise} \end{cases} \quad \check{\varphi}_S(x) = \begin{cases} \check{\varphi}_{s0}(x) & \check{a}^2 \leq x < \check{b}^2 \\ \check{\varphi}_{s1}(x) & \check{b}^2 \leq x < \check{c}^2 \\ \check{\varphi}_{s2}(x) & \check{c}^2 \leq x < \check{d}^2 \\ \check{\varphi}_{s3}(x) & \check{d}^2 \leq x < \check{e}^2 \\ \check{\varphi} & x = \check{e}^2 \\ \check{\varphi}_{s3}(x) & \check{e}^2 \leq x < \check{f}^2 \\ \check{\varphi}_{s2}(x) & \check{f}^2 \leq x < \check{g}^2 \\ \check{\varphi}_{s1}(x) & \check{g}^2 \leq x < \check{h}^2 \\ \check{\varphi}_{s0}(x) & \check{h}^2 \leq x < \check{i}^2 \\ 1 & \text{otherwise} \end{cases}$$

Where  $\check{S} = \langle [(\check{a} < \check{b} < \check{c} < \check{d} < \check{e} < \check{f} < \check{g} < \check{h} < \check{i}): \check{\theta}] [(\check{a}^1 < \check{b}^1 < \check{c}^1 < \check{d}^1 < \check{e}^1 < \check{f}^1 < \check{g}^1 < \check{h}^1 < \check{i}^1): \psi] [(\check{a}^2 < \check{b}^2 < \check{c}^2 < \check{d}^2 < \check{e}^2 < \check{f}^2 < \check{g}^2 < \check{h}^2 < \check{i}^2): \varphi] \rangle$ .

**3. In that section we investigate its representation and discuss its properties.**

### 3.1 Linear NNN with symmetry

Let  $\hat{A}_{LS} = (\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{h}, \hat{i})$  as linear NNN with these membership function:

$$Truth = \hat{T}_L(X) = \begin{cases} 0 & x < \hat{a} \\ k \left( \frac{x - \hat{a}}{\hat{b} - \hat{a}} \right) & \hat{a} < x < \hat{b} \\ k \left( \frac{x - \hat{b}}{\hat{c} - \hat{b}} \right) & \hat{b} < x < \hat{c} \\ k & \hat{c} < x < \hat{d} \\ k + (1 - k) \left( \frac{x - \hat{d}}{\hat{e} - \hat{d}} \right) & \hat{d} < x < \hat{e} \\ 1 & \hat{e} < x < \hat{f} \\ k + (1 - k) \left( \frac{\hat{g} - x}{\hat{g} - \hat{f}} \right) & \hat{f} < x < \hat{g} \\ k & \hat{g} < x < \hat{h} \\ k \left( \frac{\hat{i} - x}{\hat{i} - \hat{h}} \right) & \hat{h} < x < \hat{i} \\ 0 & x > \hat{i} \end{cases}$$

$$\begin{aligned}
 \text{Falsity} = \hat{F}_L(X) &= \begin{cases} 0 & x < \hat{a}^1 \\ k \left( \frac{x - \hat{a}^1}{\hat{b}^1 - \hat{a}^1} \right) & \hat{a}^1 < x < \hat{b}^1 \\ k \left( \frac{x - \hat{b}^1}{\hat{c}^1 - \hat{b}^1} \right) & \hat{b}^1 < x < \hat{c}^1 \\ k & \hat{c}^1 < x < \hat{d}^1 \\ k + (1 - k) \left( \frac{x - \hat{d}^1}{\hat{e}^1 - \hat{d}^1} \right) & \hat{d}^1 < x < \hat{e}^1 \\ 1 & \hat{e}^1 < x < \hat{f}^1 \\ k + (1 - k) \left( \frac{\hat{g}^1 - x}{\hat{g}^1 - \hat{f}^1} \right) & \hat{f}^1 < x < \hat{g}^1 \\ k & \hat{g}^1 < x < \hat{h}^1 \\ k \left( \frac{\hat{i}^1 - x}{\hat{i}^1 - \hat{h}^1} \right) & \hat{h}^1 < x < \hat{i}^1 \\ 1 & x > \hat{i}^1 \end{cases} \\
 \text{Indeterminacy} = \hat{I}_L(X) &= \begin{cases} 0 & x < \hat{a}^2 \\ k \left( \frac{x - \hat{a}^2}{\hat{b}^2 - \hat{a}^2} \right) & \hat{a}^2 < x < \hat{b}^2 \\ k \left( \frac{x - \hat{b}^2}{\hat{c}^2 - \hat{b}^2} \right) & \hat{b}^2 < x < \hat{c}^2 \\ k & \hat{c}^2 < x < \hat{d}^2 \\ k + (1 - k) \left( \frac{x - \hat{d}^2}{\hat{e}^2 - \hat{d}^2} \right) & \hat{d}^2 < x < \hat{e}^2 \\ 1 & \hat{e}^2 < x < \hat{f}^2 \\ k + (1 - k) \left( \frac{\hat{g}^2 - x}{\hat{g}^2 - \hat{f}^2} \right) & \hat{f}^2 < x < \hat{g}^2 \\ k & \hat{g}^2 < x < \hat{h}^2 \\ k \left( \frac{\hat{i}^2 - x}{\hat{i}^2 - \hat{h}^2} \right) & \hat{h}^2 < x < \hat{i}^2 \\ 1 & x > \hat{i}^2 \end{cases}
 \end{aligned}$$

As,  $0 < k < 1$

$$\mathcal{A}_{\alpha} = \{x \in \mathcal{X} \mid \hat{T}_L(X), \hat{F}_L(X), \hat{I}_L(X) \geq \alpha\}$$

**3.2  $\alpha$ -cut of Linear ONN with symmetry:**  $\alpha$ -cut can be express as:

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a} + \frac{\alpha}{b_1}(\hat{b} - \hat{a}) \text{ for } \alpha \in [0, b_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b} + \frac{1-\alpha}{1-b_2}(\hat{c} - \hat{b}) \text{ for } \alpha \in [b_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c} + \frac{1-\alpha}{1-b_3}(\hat{d} - \hat{c}) \text{ for } \alpha \in [b_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d} + \frac{1-\alpha}{1-b_4}(\hat{e} - \hat{d}) \text{ for } \alpha \in [b_4, 1] \\ \hat{\mathcal{A}}_{4R}(\alpha) = \hat{e} - \frac{\alpha}{b_4}(\hat{f} - \hat{e}) \text{ for } \alpha \in [0, b_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f} - \frac{\alpha}{b_3}(\hat{g} - \hat{f}) \text{ for } \alpha \in [0, b_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g} - \frac{\alpha}{b_2}(\hat{h} - \hat{g}) \text{ for } \alpha \in [0, b_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h} - \frac{\alpha}{b_1}(\hat{i} - \hat{h}) \text{ for } \alpha \in [0, b_1] \end{cases}$$

$$\text{Falsity} = \hat{F}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a}^1 + \frac{\alpha}{\hat{b}_1}(\hat{b}^1 - \hat{a}^1) \text{ for } \alpha \in [0, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b}^1 + \frac{1-\alpha}{1-\hat{b}_2}(\hat{c}^1 - \hat{b}^1) \text{ for } \alpha \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c}^1 + \frac{1-\alpha}{1-\hat{b}_3}(\hat{d}^1 - \hat{c}^1) \text{ for } \alpha \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d}^1 + \frac{1-\alpha}{1-\hat{b}_4}(\hat{e}^1 - \hat{d}^1) \text{ for } \alpha \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{e}^1 - \frac{\alpha}{\hat{b}_4}(\hat{f}^1 - \hat{e}^1) \text{ for } \alpha \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f}^1 - \frac{\alpha}{\hat{b}_3}(\hat{g}^1 - \hat{f}^1) \text{ for } \alpha \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g}^1 - \frac{\alpha}{\hat{b}_2}(\hat{h}^1 - \hat{g}^1) \text{ for } \alpha \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h}^1 - \frac{\alpha}{\hat{b}_1}(\hat{i}^1 - \hat{h}^1) \text{ for } \alpha \in [0, \hat{b}_1] \end{cases}$$

$$\text{Indeterminacy} = \hat{I}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a}^2 + \frac{\alpha}{\hat{b}_1}(\hat{b}^2 - \hat{a}^2) \text{ for } \alpha \in [0, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b}^2 + \frac{1-\alpha}{1-\hat{b}_2}(\hat{c}^2 - \hat{b}^2) \text{ for } \alpha \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c}^2 + \frac{1-\alpha}{1-\hat{b}_3}(\hat{d}^2 - \hat{c}^2) \text{ for } \alpha \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d}^2 + \frac{1-\alpha}{1-\hat{b}_4}(\hat{e}^2 - \hat{d}^2) \text{ for } \alpha \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{4R}(\alpha) = \hat{e}^2 - \frac{\alpha}{\hat{b}_4}(\hat{f}^2 - \hat{e}^2) \text{ for } \alpha \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f}^2 - \frac{\alpha}{\hat{b}_3}(\hat{g}^2 - \hat{f}^2) \text{ for } \alpha \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g}^2 - \frac{\alpha}{\hat{b}_2}(\hat{h}^2 - \hat{g}^2) \text{ for } \alpha \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h}^2 - \frac{\alpha}{\hat{b}_1}(\hat{i}^2 - \hat{h}^2) \text{ for } \alpha \in [0, \hat{b}_1] \end{cases}$$

There we have  $\hat{\mathcal{A}}_{1L}(\alpha), \hat{\mathcal{A}}_{2L}(\alpha), \hat{\mathcal{A}}_{3L}(\alpha), \hat{\mathcal{A}}_{4L}(\alpha)$  are increasing and  $\hat{\mathcal{A}}_{4R}(\alpha), \hat{\mathcal{A}}_{3R}(\alpha), \hat{\mathcal{A}}_{2R}(\alpha), \hat{\mathcal{A}}_{1R}(\alpha)$  are decreasing.

**3.3 Non-Linear Nonagonal neutrosophic numbers with symmetry:** is give as  $\hat{\mathcal{A}}_{LS} = (\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}, \hat{f}, \hat{g}, \hat{h}, \hat{i})_{(\hat{n}_1, \hat{n}_2, \hat{m}_1, \hat{m}_2)}$  as well as membership function as following:

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} 0 & x < \hat{a} \\ k \left( \frac{x - \hat{a}}{\hat{b} - \hat{a}} \right)^{n_1} & \hat{a} < x < \hat{b} \\ k \left( \frac{x - \hat{b}}{\hat{c} - \hat{b}} \right)^{n_2} & \hat{b} < x < \hat{c} \\ k & \hat{c} < x < \hat{d} \\ k + (1 - k) \left( \frac{x - \hat{d}}{\hat{e} - \hat{d}} \right)^{n_3} & \hat{d} < x < \hat{e} \\ 1 & \hat{e} < x < \hat{f} \\ k + (1 - k) \left( \frac{\hat{g} - x}{\hat{g} - \hat{f}} \right)^{m_1} & \hat{f} < x < \hat{g} \\ k & \hat{g} < x < \hat{h} \\ k \left( \frac{\hat{i} - x}{\hat{i} - \hat{h}} \right)^{m_2} & \hat{h} < x < \hat{i} \\ 0 & x > \hat{i} \end{cases}$$

$$\begin{aligned}
\text{Indeterminacy} = \hat{I}_L(X) &= \begin{cases} 0 & x < a^{\cdot 2} \\ k \left( \frac{x - a^{\cdot 2}}{b^{\cdot 2} - a^{\cdot 2}} \right)^{m_1} & a^{\cdot 2} < x < b^{\cdot 2} \\ k \left( \frac{x - b^{\cdot 2}}{c^{\cdot 2} - b^{\cdot 2}} \right)^{m_2} & b^{\cdot 2} < x < c^{\cdot 2} \\ k & c^{\cdot 2} < x < d^{\cdot 2} \\ k + (1 - k) \left( \frac{x - d^{\cdot 2}}{e^{\cdot 2} - d^{\cdot 2}} \right)^{m_3} & d^{\cdot 2} < x < e^{\cdot 2} \\ 1 & e^{\cdot 2} < x < f^{\cdot 2} \\ k + (1 - k) \left( \frac{g^{\cdot 2} - x}{g^{\cdot 2} - f^{\cdot 2}} \right)^{n_1} & f^{\cdot 2} < x < g^{\cdot 2} \\ k & g^{\cdot 2} < x < h^{\cdot 2} \\ k \left( \frac{i^{\cdot 2} - x}{i^{\cdot 2} - h^{\cdot 2}} \right)^{n_2} & h^{\cdot 2} < x < i^{\cdot 2} \\ 1 & x > i^{\cdot 2} \end{cases} \quad \text{As, } 0 < k < 1 \\
\text{Falsity} = \hat{F}_L(X) &= \begin{cases} 0 & x < a^{\cdot 1} \\ k \left( \frac{x - a^{\cdot 1}}{b^{\cdot 1} - a^{\cdot 1}} \right)^{m_1} & a^{\cdot 1} < x < b^{\cdot 1} \\ k \left( \frac{x - b^{\cdot 1}}{c^{\cdot 1} - b^{\cdot 1}} \right)^{m_2} & b^{\cdot 1} < x < c^{\cdot 1} \\ k & c^{\cdot 1} < x < d^{\cdot 1} \\ k + (1 - k) \left( \frac{x - d^{\cdot 1}}{e^{\cdot 1} - d^{\cdot 1}} \right)^{m_3} & d^{\cdot 1} < x < e^{\cdot 1} \\ 1 & e^{\cdot 1} < x < f^{\cdot 1} \\ k + (1 - k) \left( \frac{g^{\cdot 1} - x}{g^{\cdot 1} - f^{\cdot 1}} \right)^{n_1} & f^{\cdot 1} < x < g^{\cdot 1} \\ k & g^{\cdot 1} < x < h^{\cdot 1} \\ k \left( \frac{i^{\cdot 1} - x}{i^{\cdot 1} - h^{\cdot 1}} \right)^{n_2} & h^{\cdot 1} < x < i^{\cdot 1} \\ 1 & x > i^{\cdot 1} \end{cases} \quad , \mathcal{A}_{\hat{\alpha}} = \{x \in \mathcal{X} \mid \hat{T}_L(X), \hat{F}_L(X), \hat{I}_L(X) \geq \hat{\alpha}\}
\end{aligned}$$

### 3.4 $\alpha$ - cut of Non-Linear NFN with symmetry:

$\alpha$  - cut of non-LONNS can be express by  $\mathcal{A}_{\hat{\alpha}} = \{x \in \mathcal{X} \mid \hat{T}_L(X), \hat{F}_L(X), \hat{I}_L(X) \geq \hat{\alpha}\}$

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\hat{\alpha}) = \hat{a}^{\cdot} + \left( \frac{\hat{\alpha}}{\hat{b}_1^{\cdot}} \right)^{\hat{n}_1} (\hat{b}^{\cdot} - \hat{a}^{\cdot}) \text{ for } \hat{\alpha} \in [\hat{0}, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\hat{\alpha}) = \hat{b}^{\cdot} + \left( \frac{1 - \hat{\alpha}}{1 - \hat{b}_2^{\cdot}} \right)^{\hat{n}_2} (\hat{c}^{\cdot} - \hat{b}^{\cdot}) \text{ for } \hat{\alpha} \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\hat{\alpha}) = \hat{c}^{\cdot} + \left( \frac{1 - \hat{\alpha}}{1 - \hat{b}_3^{\cdot}} \right)^{\hat{n}_3} (\hat{d}^{\cdot} - \hat{c}^{\cdot}) \text{ for } \hat{\alpha} \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\hat{\alpha}) = \hat{d}^{\cdot} + \left( \frac{1 - \hat{\alpha}}{1 - \hat{b}_4^{\cdot}} \right)^{\hat{n}_4} (\hat{e}^{\cdot} - \hat{d}^{\cdot}) \text{ for } \hat{\alpha} \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{4R}(\hat{\alpha}) = \hat{e}^{\cdot} - \left( \frac{\hat{\alpha}}{\hat{b}_4^{\cdot}} \right)^{\hat{m}_1} (\hat{f}^{\cdot} - \hat{e}^{\cdot}) \text{ for } \hat{\alpha} \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\hat{\alpha}) = \hat{f}^{\cdot} - \left( \frac{\hat{\alpha}}{\hat{b}_3^{\cdot}} \right)^{\hat{m}_2} (\hat{g}^{\cdot} - \hat{f}^{\cdot}) \text{ for } \hat{\alpha} \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\hat{\alpha}) = \hat{g}^{\cdot} - \left( \frac{\hat{\alpha}}{\hat{b}_2^{\cdot}} \right)^{\hat{m}_3} (\hat{h}^{\cdot} - \hat{g}^{\cdot}) \text{ for } \hat{\alpha} \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\hat{\alpha}) = \hat{h}^{\cdot} - \left( \frac{\hat{\alpha}}{\hat{b}_1^{\cdot}} \right)^{\hat{m}_4} (\hat{i}^{\cdot} - \hat{h}^{\cdot}) \text{ for } \hat{\alpha} \in [0, \hat{b}_1] \end{cases}$$

$$\text{Indeterminacy} = \hat{I}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a}^2 + \left(\frac{\alpha}{\hat{b}_1}\right)^{\hat{m}_1} (\hat{b}^2 - \hat{a}^2) \text{ for } \alpha \in [\hat{0}, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b}^2 + \left(\frac{1-\alpha}{1-\hat{b}_2}\right)^{\hat{m}_2} (\hat{c}^2 - \hat{b}^2) \text{ for } \alpha \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c}^2 + \left(\frac{1-\alpha}{1-\hat{b}_3}\right)^{\hat{m}_3} (\hat{d}^2 - \hat{c}^2) \text{ for } \alpha \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d}^2 + \left(\frac{1-\alpha}{1-\hat{b}_4}\right)^{\hat{m}_4} (\hat{e}^2 - \hat{d}^2) \text{ for } \alpha \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{e}^2 - \left(\frac{\alpha}{\hat{b}_4}\right)^{\hat{n}_1} (\hat{f}^2 - \hat{e}^2) \text{ for } \alpha \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f}^2 - \left(\frac{\alpha}{\hat{b}_3}\right)^{\hat{n}_2} (\hat{g}^2 - \hat{f}^2) \text{ for } \alpha \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g}^2 - \left(\frac{\alpha}{\hat{b}_2}\right)^{\hat{n}_3} (\hat{h}^2 - \hat{g}^2) \text{ for } \alpha \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h}^2 - \left(\frac{\alpha}{\hat{b}_1}\right)^{\hat{n}_4} (\hat{i}^2 - \hat{h}^2) \text{ for } \alpha \in [0, \hat{b}_1] \end{cases}$$

$$\text{Falsity} = \hat{F}_L(X) = \begin{cases} \hat{\mathcal{A}}_{1L}(\alpha) = \hat{a}^1 + \left(\frac{\alpha}{\hat{b}_1}\right)^{\hat{m}_1} (\hat{b}^1 - \hat{a}^1) \text{ for } \alpha \in [\hat{0}, \hat{b}_1] \\ \hat{\mathcal{A}}_{2L}(\alpha) = \hat{b}^1 + \left(\frac{1-\alpha}{1-\hat{b}_2}\right)^{\hat{m}_2} (\hat{c}^1 - \hat{b}^1) \text{ for } \alpha \in [\hat{b}_2, 1] \\ \hat{\mathcal{A}}_{3L}(\alpha) = \hat{c}^1 + \left(\frac{1-\alpha}{1-\hat{b}_3}\right)^{\hat{m}_3} (\hat{d}^1 - \hat{c}^1) \text{ for } \alpha \in [\hat{b}_3, 1] \\ \hat{\mathcal{A}}_{4L}(\alpha) = \hat{d}^1 + \left(\frac{1-\alpha}{1-\hat{b}_4}\right)^{\hat{m}_4} (\hat{e}^1 - \hat{d}^1) \text{ for } \alpha \in [\hat{b}_4, 1] \\ \hat{\mathcal{A}}_{4R}(\alpha) = \hat{e}^1 - \left(\frac{\alpha}{\hat{b}_4}\right)^{\hat{n}_1} (\hat{f}^1 - \hat{e}^1) \text{ for } \alpha \in [0, \hat{b}_4] \\ \hat{\mathcal{A}}_{3R}(\alpha) = \hat{f}^1 - \left(\frac{\alpha}{\hat{b}_3}\right)^{\hat{n}_2} (\hat{g}^1 - \hat{f}^1) \text{ for } \alpha \in [0, \hat{b}_3] \\ \hat{\mathcal{A}}_{2R}(\alpha) = \hat{g}^1 - \left(\frac{\alpha}{\hat{b}_2}\right)^{\hat{n}_3} (\hat{h}^1 - \hat{g}^1) \text{ for } \alpha \in [0, \hat{b}_2] \\ \hat{\mathcal{A}}_{1R}(\alpha) = \hat{h}^1 - \left(\frac{\alpha}{\hat{b}_1}\right)^{\hat{n}_4} (\hat{i}^1 - \hat{h}^1) \text{ for } \alpha \in [0, \hat{b}_1] \end{cases}$$

The function which are increasing are  $\hat{\mathcal{A}}_{1L}(\alpha), \hat{\mathcal{A}}_{2L}(\alpha), \hat{\mathcal{A}}_{3L}(\alpha), \hat{\mathcal{A}}_{4L}(\alpha)$  with respect to  $\alpha$  and  $\hat{\mathcal{A}}_{4R}(\alpha), \hat{\mathcal{A}}_{3R}(\alpha), \hat{\mathcal{A}}_{2R}(\alpha), \hat{\mathcal{A}}_{1R}(\alpha)$  are decreasing with respect to  $\alpha$ .

$$\text{Truth} = \hat{T}_L(X) = \begin{cases} 0 & x < \hat{a} \\ p \left( \frac{x-\hat{a}}{\hat{b}-\hat{a}} \right)^{n_1} & \hat{a} < x < \hat{b} \\ p \left( \frac{x-\hat{b}}{\hat{c}-\hat{b}} \right)^{n_2} & \hat{b} < x < \hat{c} \\ k & \hat{c} < x < \hat{d} \\ k - (k-p) \left( \frac{x-\hat{d}}{\hat{e}-\hat{d}} \right)^{n_3} & \hat{d} < x < \hat{e} \\ 1 & \hat{e} < x < \hat{f} \\ k - (k-r) \left( \frac{\hat{g}-x}{\hat{g}-\hat{f}} \right)^{m_1} & \hat{f} < x < \hat{g} \\ k & \hat{g} < x < \hat{h} \\ r \left( \frac{\hat{i}-x}{\hat{i}-\hat{h}} \right)^{m_2} & \hat{h} < x < \hat{i} \\ 0 & x > \hat{i} \end{cases}$$

$$\begin{aligned}
\text{Indeterminacy} = \hat{I}_L(X) &= \begin{cases} 0 & x < a^{\cdot 2} \\ y \left( \frac{x - a^{\cdot 2}}{b^{\cdot 2} - a^{\cdot 2}} \right)^{m_1} & a^{\cdot 2} < x < b^{\cdot 2} \\ y \left( \frac{x - b^{\cdot 2}}{c^{\cdot 2} - b^{\cdot 2}} \right)^{m_2} & b^{\cdot 2} < x < c^{\cdot 2} \\ X & c^{\cdot 2} < x < d^{\cdot 2} \\ X - (X - Y) \left( \frac{x - d^{\cdot 2}}{e^{\cdot 2} - d^{\cdot 2}} \right)^{m_3} & d^{\cdot 2} < x < e^{\cdot 2} \\ 1 & e^{\cdot 2} < x < f^{\cdot 2} \\ x - (X - z) \left( \frac{g^{\cdot 2} - x}{g^{\cdot 2} - f^{\cdot 2}} \right)^{n_1} & f^{\cdot 2} < x < g^{\cdot 2} \\ z & g^{\cdot 2} < x < h^{\cdot 2} \\ z \left( \frac{i^{\cdot 2} - x}{i^{\cdot 2} - h^{\cdot 2}} \right)^{n_2} & h^{\cdot 2} < x < i^{\cdot 2} \\ 1 & x > i^{\cdot 2} \end{cases} \\
\text{Falsity} = \hat{F}_L(X) &= \begin{cases} 0 & x < a^{\cdot 1} \\ q \left( \frac{x - a^{\cdot 1}}{b^{\cdot 1} - a^{\cdot 1}} \right)^{m_1} & a^{\cdot 1} < x < b^{\cdot 1} \\ q \left( \frac{x - b^{\cdot 1}}{c^{\cdot 1} - b^{\cdot 1}} \right)^{m_2} & b^{\cdot 1} < x < c^{\cdot 1} \\ k & c^{\cdot 1} < x < d^{\cdot 1} \\ w - (w - q) \left( \frac{x - d^{\cdot 1}}{e^{\cdot 1} - d^{\cdot 1}} \right)^{m_3} & d^{\cdot 1} < x < e^{\cdot 1} \\ 1 & e^{\cdot 1} < x < f^{\cdot 1} \\ w - (w - s) \left( \frac{g^{\cdot 1} - x}{g^{\cdot 1} - f^{\cdot 1}} \right)^{n_1} & f^{\cdot 1} < x < g^{\cdot 1} \\ w & g^{\cdot 1} < x < h^{\cdot 1} \\ s \left( \frac{i^{\cdot 1} - x}{i^{\cdot 1} - h^{\cdot 1}} \right)^{n_2} & h^{\cdot 1} < x < i^{\cdot 1} \\ 1 & x > i^{\cdot 1} \end{cases} \quad \text{As, } 0 < k < 1
\end{aligned}$$

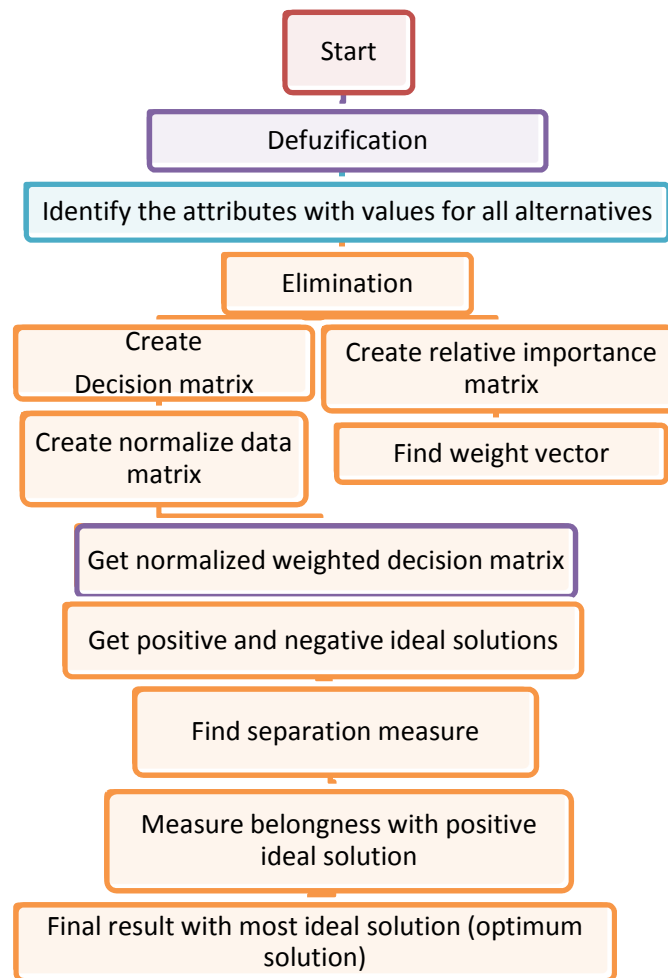
#### 4. TOPSIS Technique

TOPSIS is a technique used for order performance by the help of similarity to the ideal solution. This idea is given by Hwang and Yoon. This is one of the most common technique for daily life decision making problem. This method is really close to the concept of PIS and NIS. Other methods like VIKOR also suitable for that kind of situation. We defuzzied the octagonal Fuzzy number by

$$\hat{D}^{TNO_N} = \left( \frac{a^{\cdot 2} + b^{\cdot 2} + c^{\cdot 2} + d^{\cdot 2} + e^{\cdot 2} + f^{\cdot 2} + g^{\cdot 2} + h^{\cdot 2} + i^{\cdot 2}}{9} \right), \quad \hat{D}^{INO_N} = \left( \frac{a^{\cdot 1} + b^{\cdot 1} + c^{\cdot 1} + d^{\cdot 1} + e^{\cdot 1} + f^{\cdot 1} + g^{\cdot 1} + h^{\cdot 1} + i^{\cdot 1}}{9} \right) \text{ and}$$

$$\hat{D}^{FNO_N} = \left( \frac{a^{\cdot 2} + b^{\cdot 2} + c^{\cdot 2} + d^{\cdot 2} + e^{\cdot 2} + f^{\cdot 2} + g^{\cdot 2} + h^{\cdot 2} + i^{\cdot 2}}{9} \right)$$

$$d(\bar{x}, \bar{y}) = \sqrt{\frac{1}{3} (a_1 - \hat{a}_2)^2 + (b_1 - \hat{b}_2)^2 + (c_1 - \hat{c}_2)^2}$$



**Step 1:** We will assigned the rating to the criteria and by given alternatives. Here we have a supposition that we have a decision group with K members. The alternative is shown as  $\mathcal{A}_i$  with respect to criteria  $\mathcal{C}_j$  is denoted  $\bar{x}_{ij}^k = (\bar{a}_{ij}^k, b_{ij}^k, \bar{c}_{ij}^k)$  and the weight criteria is given as  $\bar{w}_j^k = (\bar{w}_{j1}^k, w_{j2}^k, \bar{w}_{j3}^k)$ .

**Step 2:** we will make a relation between fuzzy rating with alternatives and fuzzy weighting with criteria.

Linguistic value	Nonagonal FN $\bar{a}_{ij}$
<b>Absolutely Recommended</b>	(7,8,9,8,6,8,6,7,9)
<b>Strongly Recommended</b>	(6,7,8,7,8,6,5,7,8)
<b>Recommended</b>	(5,6,7,7,7,6,8,7,7)
<b>May be Recommended</b>	(4,5,7,4,5,7,4,7,5)
<b>Weakly Recommended</b>	(2,3,5,5,6,3,4,3,4)
<b>Rarely Recommended</b>	(1,2,4,3,2,4,4,3,5)
<b>Not Sure to be Recommended</b>	(1,1,3,2,2,1,3,2,3)

Fuzzy Rating as  $\bar{x}_{ij}^k = (a_{ij}, b_{ij}, \bar{c}_{ij})$  of  $i^{th}$  alternative with respect to  $j^{th}$  criteria.

$$\bar{a}_{ij} = \min_k \{a_{ij}^k\}, \quad \bar{b}_{ij} = \frac{1}{K} \sum_{k=1}^K b_{ij}^k, \quad c_{ij} = \max_k \{c_{ij}^k\}.$$

The weight as  $\bar{w}_j = (\bar{w}_{j1}, \bar{w}_{j2}, \bar{w}_{j3})$  for criteria  $\bar{C}_j$  are given by these formulas:

$$\bar{w}_{j1} = \min_k \{w_{j1}^k\}, \quad \bar{w}_{j2} = \frac{1}{K} \sum_{k=1}^K w_{j2}^k, \quad w_{j3} = \max_k \{w_{j3}^k\}.$$

Step 3: The normalized fuzzy decision matrix as  $\bar{R} = [\bar{r}_{ij}]$ ,

$$\bar{r}_{ij} = \left( \frac{\bar{a}_{ij}}{c_j^*}, \frac{\bar{b}_{ij}}{c_j^*}, \frac{\bar{c}_{ij}}{c_j^*} \right) \text{ and } c_j^* = \max_i \{c_{ij}\} \text{ beneficial criteria}$$

$$\bar{r}_{ij} = \left( \frac{\bar{a}_{ij}}{c_{ij}^-}, \frac{\bar{a}_{ij}}{b_{ij}^-}, \frac{\bar{a}_{ij}}{a_{ij}^-} \right) \text{ and } c_j^- = \min_i \{a_{ij}\} \text{ non beneficial criteria.}$$

**Step 4:** Now weighted normalized fuzzy decision matrix will compute.

**Step 5:** Now compute the fuzzy negative ideal solution FNIS and fuzzy positive ideal solution FPIS.

**Step 6:** get a solution of each distance of alternative to FPIS and FNIS.

$$\bar{d}_i^+ = \sum_{j=1}^n \bar{d}(v_{ij}, \bar{v}_j^+), \quad \bar{d}_i^- = \sum_{j=1}^n \bar{d}(v_{ij}, \bar{v}_j^-) \text{ be the distance.}$$

$$\text{Step 7: Closeness Coefficient } \bar{CC}_i = \frac{\bar{d}_i^-}{\bar{d}_i^- + \bar{d}_i^+}$$

**Step 8:** Alternatives will be ranked.

**5 Case Study:** We will check the productivity and flexibility of this proposed TOPSIS method, here we have most useful real-life issue. Suppose that, A person is going to buy a car, here a lot of possibilities available and decision based on three different criteria. We have to select best one.

**Numerical problem:** Here U is the universe. Person is going to buy a car, So, give a look on possible options, mention in Table 1. Three different brands ( $\bar{A}$ ,  $\bar{B}$ ,  $\bar{C}$ ) applied for this opportunity, with different opportunity. Choice parameter are  $\{\bar{C}_1, \bar{C}_2, \bar{C}_3\}$  and detailed mention in table 1.

$\bar{A}$	$\bar{B}$	$\bar{C}$
$\{\bar{C}_1(0.71, 0.35, 0.71, 0.77, 0.31, 0.73, 0.67, 0.61, 0.3)$ $(0.93, 0.43, 0.93, 0.88, 0.84, 0.99, 0.96, 0.90, 0.4)$ $(0.86, 0.95, 0.89, 0.97, 0.94, 0.93, 0.75, 0.81, 0.99)\}$ $\{\bar{C}_2(0.85, 0.75, 0.96, 0.54, 0.83, 0.75, 0.63, 0.56, 0.97)$ $(0.85, 0.45, 0.65, 0.38, 0.78, 0.79, 0.57, 0.13, 0.9)$ $(0.96, 0.89, 0.98, 0.99, 0.97, 0.96, 0.97, 0.95, 0.11)\}$ $\{\bar{C}_3(0.74, 0.83, 0.94, 0.75, 0.96, 0.34, 0.75, 0.69, 0.95)$ $(0.35, 0.46, 0.98, 0.59, 0.65, 0.71, 0.74, 0.76, 0.25)$ $(0.84, 0.73, 0.75, 0.85, 0.98, 0.74, 0.86, 0.84, 0.91)\}$	$\{\bar{C}_1(0.63, 0.73, 0.36, 0.94, 0.85, 0.86, 0.84, 0.85, 0.89)$ $(0.33, 0.46, 0.79, 0.91, 0.79, 0.75, 0.79, 0.74, 0.96)$ $(0.98, 0.93, 0.95, 0.85, 0.97, 0.98, 0.74, 0.86, 0.90)\}$ $\{\bar{C}_2(0.66, 0.49, 0.68, 0.99, 0.67, 0.11, 0.56, 0.87, 0.95)$ $(0.93, 0.73, 0.83, 0.68, 0.84, 0.79, 0.45, 0.76, 0.70)$ $(0.85, 0.95, 0.96, 0.64, 0.93, 0.75, 0.73, 0.55, 0.96)\}$ $\{\bar{C}_3(0.94, 0.93, 0.84, 0.95, 0.96, 0.84, 0.95, 0.91, 0.99)$ $(0.28, 0.36, 0.58, 0.25, 0.65, 0.61, 0.54, 0.26, 0.88)$ $(0.98, 0.93, 0.95, 0.95, 0.78, 0.94, 0.96, 0.94, 0.91)\}$	$\{\bar{C}_1(0.63, 0.83, 0.83, 0.66, 0.85, 0.44, 0.85, 0.63, 0.98)$ $(0.76, 0.75, 0.69, 0.84, 0.94, 0.97, 0.63, 0.75, 0.63)\}$ $(0.88, 0.93, 0.95, 0.95, 0.77, 0.98, 0.84, 0.86, 0.90)$ $\{\bar{C}_2(0.98, 0.69, 0.98, 0.88, 0.79, 0.97, 0.96, 0.97, 0.85)$ $(0.75, 0.45, 0.55, 0.45, 0.28, 0.78, 0.59, 0.67, 0.23)$ $(0.89, 0.95, 0.86, 0.94, 0.93, 0.95, 0.91, 0.99, 0.99)\}$ $\{\bar{C}_3(0.84, 0.96, 0.33, 0.35, 0.52, 0.97, 0.93, 0.21, 0)$ $(0.45, 0.95, 0.54, 0.78, 0.39, 0.75, 0.61, 0.44, 0.76)$ $(0.21, 0.74, 0.33, 0.64, 0.85, 0.76, 0.34, 0.65, 0.99)\}$

[Now, this given matrix depend on  $(\bar{C}_1, \bar{C}_2, \bar{C}_3)$  as row and  $(\bar{A}, \bar{B}, \bar{C})$  as column]

**STEP 1.** We defuzzified the Octagonal Fuzzy number by

$$\bar{D}^{TNO} = \left( \frac{\bar{a}^1 + \bar{b}^1 + \bar{c}^1 + \bar{d}^1 + \bar{e}^1 + \bar{f}^1 + \bar{g}^1 + \bar{h}^1 + \bar{i}^1}{9} \right), \quad \bar{D}^{INO} = \left( \frac{\bar{a}^1 + \bar{b}^1 + \bar{c}^1 + \bar{d}^1 + \bar{e}^1 + \bar{f}^1 + \bar{g}^1 + \bar{h}^1 + \bar{i}^1}{9} \right) \text{ and}$$

$$\bar{D}^{FNO} = \left( \frac{\bar{a}^2 + \bar{b}^2 + \bar{c}^2 + \bar{d}^2 + \bar{e}^2 + \bar{f}^2 + \bar{g}^2 + \bar{h}^2 + \bar{i}^2}{9} \right)$$

By applying these formulas, Neutrosophic soft matrix as following:

Criteria	$\hat{A}$	$\hat{B}$	$\hat{C}$
$\hat{C}_1$	(0.6, 0.8, 0.9)	(0.8, 0.7, 0.9)	(0.7, 0.8, 0.9)
$\hat{C}_2$	(0.7, 0.6, 0.9)	(0.7, 0.8, 0.8)	(0.8, 0.5, 0.9)
$\hat{C}_3$	(0.8, 0.6, 0.8)	(0.9, 0.5, 0.9)	(0.6, 0.6, 0.6)

**STEP 2:** For normalized aggregate fuzzy decision matrix.

$$\tilde{r}_{ij} = \left( \frac{\tilde{a}_{ij}}{\tilde{c}_{ij}}, \frac{\tilde{b}_{ij}}{\tilde{c}_{ij}}, \frac{\tilde{c}_{ij}}{\tilde{c}_{ij}} \right)$$

Criteria	$\hat{A}$	$\hat{B}$	$\hat{C}$
$\hat{C}_1$	(0.7, 0.9, 1.0)	(0.9, 0.8, 1.0)	(0.8, 0.9, 1.0)
$\hat{C}_2$	(0.8, 0.7, 1.0)	(0.9, 1.0, 1.0)	(0.9, 0.5, 1.0)
$\hat{C}_3$	(1.0, 0.7, 1.0)	(1.0, 0.5, 1.0)	(1.0, 1.0, 1.0)

Then for criteria weighting there is a aggregate decision matrix

$$\bar{W}_1 = (0.6, 0.6, 0.7) \quad , \quad \bar{W}_2 = (0.1, 0.2, 0.4) \quad , \quad \bar{W}_3 = (0.3, 0.5, 0.5)$$

**STEP 3:**  $\tilde{p}_{ij} = \tilde{r}_{ij}$  will multiply by  $\bar{w}_j$  as well as weighted normalized fuzzy decision matrix.

Criteria	$\hat{A}$	$\hat{B}$	$\hat{C}$
$\hat{C}_1$	(0.4, 0.5, 0.7)	(0.09, 0.1, 0.4)	(0.2, 0.4, 0.5)
$\hat{C}_2$	(0.4, 0.6, 0.7)	(0.09, 0.2, 0.4)	(0.2, 0.2, 0.5)
$\hat{C}_3$	(0.6, 0.4, 0.7)	(0.1, 0.1, 0.4)	(0.3, 0.5, 0.5)

**STEP 4:** Find  $\hat{F}NIS$  and  $\hat{F}PIS$

$$\hat{\mathcal{A}}^+ = (\hat{\mathcal{P}}_1^+, \hat{\mathcal{P}}_2^+, \hat{\mathcal{P}}_3^+ \dots \hat{\mathcal{P}}_n^+)$$

$$\hat{\mathcal{P}}_j^+ = \max (\hat{\mathcal{P}}_{ij3}) \quad i=1,2,\dots,m \quad , \quad j=1,2,3,\dots,n$$

$$\hat{\mathcal{A}}^- = (\hat{\mathcal{P}}_1^-, \hat{\mathcal{P}}_2^-, \hat{\mathcal{P}}_3^- \dots \hat{\mathcal{P}}_n^-)$$

$$\hat{\mathcal{P}}_j^- = \min (\hat{\mathcal{P}}_{ij3}) \quad i=1,2,\dots,m \quad , \quad j=1,2,3,\dots,n$$

$$\hat{\mathcal{A}}^+ = \hat{\mathcal{P}}_1^+(0.7, 0.4, 0.5), \hat{\mathcal{P}}_2^+(0.7, 0.4, 0.5), \hat{\mathcal{P}}_3^+(0.7, 0.4, 0.5)$$

$$\hat{\mathcal{A}}^- = \hat{\mathcal{P}}_1^-(0.4, 0.09, 0.2), \hat{\mathcal{P}}_2^-(0.4, 0.09, 0.2), \hat{\mathcal{P}}_3^-(0.4, 0.1, 0.3)$$

$$\text{Now by } \hat{\mathcal{A}}(\hat{\mathcal{X}}, \hat{\mathcal{Y}}) = \sqrt{\frac{1}{3} (\hat{\alpha}_1 - \hat{\alpha}_2)^2 + (\hat{\mathbf{b}}_1 - \hat{\mathbf{b}}_2)^2 + (\hat{\mathbf{c}}_1 - \hat{\mathbf{c}}_2)^2}$$

**STEP 5 to STEP 8 mentioned below**

**Table (Positive ideal solution)**

Criteria	$\hat{A}$	$\hat{B}$	$\hat{C}$
$\hat{C}_1$	0.4472	0.2733	0.2886
$\hat{C}_2$	0.3316	0.2171	0.3511
$\hat{C}_3$	0.2081	0.3690	0.2516

FNIS Table (Negative ideal solution)

Criteria	$\hat{A}$	$\hat{B}$	$\hat{C}$
$\hat{C}_1$	0.2466	0.2685	0.2465
$\hat{C}_2$	0.3142	0.2900	0.3397
$\hat{C}_3$	0.3131	0.2000	0.2509

Now estimate the distance between each weighted alternative.

$$d_i^* = \sum_{j=1}^n d(v_{ij}, v_j^*), d_i^- = \sum_{j=1}^n d(v_{ij}, v_j^-)$$

$$d_1^* = 0.9869 \quad d_1^- = 0.8739$$

$$d_2^* = 0.8594 \quad d_2^- = 0.7585$$

$$d_3^* = 0.8913 \quad d_3^- = 0.8371$$

#### Closeness coefficient

$$\hat{C}\hat{C}_i = \frac{d_i^-}{d_i^- + d_i^*}$$

$$\hat{C}\hat{C}_1 = \frac{0.8739}{0.8739 + 0.9869} = 0.4696,$$

$$\hat{C}\hat{C}_2 = \frac{0.7585}{0.7585 + 0.8594} = 0.4688$$

$$\hat{C}\hat{C}_3 = \frac{0.8371}{0.8371 + 0.8913} = 0.4578$$

Strategy	Results	Rank
$\hat{C}\hat{C}_1$	0.4696	1
$\hat{C}\hat{C}_2$	0.4688	2
$\hat{C}\hat{C}_3$	0.4578	3

#### Clearly

$\hat{A} > \hat{B} > \hat{C}$ . The best car for this person is  $\hat{A}$ .

#### 6 Conclusion:

In this current article, we present type of nonagonal neutrosophic number (Linear, Non-Linear, Symmetric, Asymmetric) as well as their  $\tilde{\alpha}$  – cuts also proposed. Nonagonal neutrosophic number will be really useful in the field of multi-criteria decision making MCDM problems of daily life as well as it can deal more fluctuations. To estimate reliability and productivity, we also present a daily life problem and solved it with TOPSIS technique of MCDM. At the very first we convert nonagonal to fuzzy using accuracy function and then we use it in existing method. Further, we will present aggregate operators of nonagonal neutrosophic number as well as matrix notation with operations.

## Acknowledgement

We are really thankful to Editor-in-chief and the referees for they motivational, productive and valuable comments and thoughts for improving this current article.

**Funding:** “ we do not receive any external funding.”

**Conflict of Interest:** “ Declare no conflict of interest.”

## References

- [1] Molodtsov, D. *Soft set theory - First results, Computers and mathematics with applications*. 37, pp.19-31,1999.
- [2] Zadeh, L.A. Fuzzy sets. *Inf. Control* , 8, pp.338–353, 1965.
- [3] Chang, S.S.L.; Zadeh, L.A. On fuzzy mappings and control. *IEEE Trans. Syst. Man Cybern*, 2, pp.30–34,1972.
- [4] Dubois, D.; Prade, H. Operations on fuzzy numbers. *Int. J. Syst. Sci*, 9, pp.613–626, 1978.
- [5] Atanassov, K.T. *Intuitionistic Fuzzy Sets*; VII ITKR's Session: Sofia, Bulgarian, 1983.
- [6] Smarandache, F. *A Unifying Field in Logics Neutrosophy: Neutrosophic Probability*; American Research Press: Rehoboth, DE, USA, 1998.
- [7] F. Smarandache, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, *Inter. J. Pure Appl. Math.* Vol.24 , pp. 287–297, 2005.
- [8] Chakraborty, A.; Mondal, S. P.; Ahmadian, A.; Senu, N.; Alam, S.; and Salahshour, S.; Different Forms of Triangular Neutrosophic Numbers, De-Neutrosophication Techniques, and their Applications, *Symmetry*, vol 10, 327, 2018. ; doi:10.3390/sym10080327.
- [9] Chakraborty. A., Sankar P. Mondal, Shariful A. Ali A., Norazak S., Debashis De. and Soheil S., The Pentagonal Fuzzy Number:Its Different Representations, Properties, Ranking, Defuzzification and Application in Game Problems, *Symmetry*, vol 10, pp 248, 2018.
- [10] Saqlain. M and Florentin S. Octagonal Neutrosophic Number: Its Different Representations, Properties, Graphs and De-Neutrosophication, *International Journal of Neutrosophic Science*, vol. 8, no. 1, pp. 19–33, 2020.
- [11] Saqlain.M , A. Hamza, and S. Farooq, “Linear and Non-Linear Octagonal Neutrosophic Numbers: Its Representation,  $\alpha$ -Cut and Applications,” *International Journal of Neutrosophic Science*, vol. 3, no. 1, pp. 29–43, 2020.
- [12] Saqlain.M, A. Hamza, and M. Saeed, R. M. Zulfarnain, Aggregate, Arithmetic and Geometric Operators of Octagonal Neutrosophic Numbers and Its Application in Multi-Criteria Decision-Making Problems, *Springer Book Series* 2020.
- [13] Dubois, D.; Prade, H.Operations on fuzzy numbers. *Int. J. Syst. Sci.* vol 9, pp. 613–626, 1978.
- [14] Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy sets Syst.*, vol 20, pp. 87–96, 1986.

- [15] Smarandache, F. A Unifying Field in Logics Neutrosophy: Neutrosophic Probability, Set and Logic, 3rd ed.; American Research Press: Washington, DC, USA, 2003.
- [16] Wang, H.B.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single Valued Neutrosophic Sets. *Tech. Sci. Appl. Math.* 2010.
- [17] Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Int. Fuzzy Syst.*, vol 26, pp. 2459–2466, 2014.
- [18] Wang, J.Q.; Peng, J.J.; Zhang, H.Y.; Liu, T.; Chen, X.H. An uncertain linguistic multi-criteria group decision-making method based on a cloud model. *Group Decis. Negot.*, vol 24, pp. 171–192, 2015.
- [19] Peng, J.J.; Wang, J.Q.; Wu, X.H.; Zhang, H.Y.; Chen, X.H. The fuzzy cross-entropy for intuitionistic hesitant fuzzy sets and their application in multi-criteria decision-making. *Int. J. Syst. Sci.* vol 46, pp. 2335–2350, 2015.
- [20] Peng, J.J.; Wang, J.Q.; Wang, J.; Zhang, H.Y.; Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision making problems. *Int. J. Syst. Sci.*, vol 47, pp. 2342–2358, 2016.
- [21] Deli I, Broumi S. Neutrosophic soft matrices and NSM decision making, *Journal of Intelligent and Fuzzy System* vol 28, pp.2233–2241, 2015.
- [22] Ma YX, Wang JQ, Wang J, Wu XH. An interval neutrosophic linguistic multi-criteria group decision-making the method and its application in selecting medical treatment options, *Neural Computer Application*. DOI:10.1007/s00521-016-2203-1. 2016.
- [23] Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. An approach of the TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, vol 77, pp. 438-452, 2019.
- [24] Abdel-Baset, M., Chang, V., Gamal, A., & Smarandache, F. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in the importing field. *Computers in Industry*, vol 106, pp. 94-110, 2019.
- [25] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A group decision-making framework based on the neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, vol 43, issue 2, pp. 38-43, 2019.
- [26] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, pp.1-22, 2018.
- [27] Nabeeh, N. A., Smarandache, F., Abdel-Basset, M., El-Ghareeb, H. A., & Aboelfetouh, A. An Integrated Neutrosophic-TOPSIS Approach and Its Application to Personnel Selection: A New Trend in Brain Processing and Analysis. *IEEE Access*, vol 7, pp. 29734-29744. 2019.
- [28] Smarandache, F.” Neutrosophy. Neutrosophic probability, set, and logic, ProQuest Information & Learning, Arbor, Michigan, USA, 1998.
- [29] Abdel-Baset, M., Chang, V., & Gamal, A. Evaluation of the green supply chain management practices: A novel neutrosophic approach. *Computers in Industry*, vol 108, pp. 210-220, 2019.

- [30] Abdel-Basset, M., Saleh, M., Gamal, A., & Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing*, vol 77, pp. 438-452, 2019.
- [31] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. A group decision making framework based on neutrosophic TOPSIS approach for smart medical device selection. *Journal of medical systems*, vol 43(2), pp. 38, 2019.
- [32] Abdel-Basset, M., Atef, A., & Smarandache, F. A hybrid Neutrosophic multiple criteria group decision making approach for project selection. *Cognitive Systems Research*, vol 57, pp. 216-227, 2019.
- [33] Abdel-Basset, Mohamed, Mumtaz Ali, and Asma Atef. "Resource levelling problem in construction projects under neutrosophic environment." *The Journal of Supercomputing*, pp.1-25, 2019.
- [34] Saqlain M, Sana M, Jafar N, Saeed. M, Said. B, Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets, *Neutrosophic Sets and Systems (NSS)*, vol 32, pp. 317-329, 2020.
- [35] S. Pramanik, P. P. Dey and B. C. Giri, "TOPSIS for single valued neutrosophic soft expert set based multiattribute decision making problems," *Neutrosophic Sets and Systems*, vol 10, pp. 88-95, 2015.
- [36] Saqlain. M., Jafar. N. and Riffat. A., "Smart phone selection by consumers' in Pakistan: FMCGDM fuzzy multiple criteria group decision making approach," *Gomal University Journal of Research*, vol 34(1), pp. 27-31, 2018.
- [37] Saqlain. M, Jafar.N. M, and Muniba. K, "Change in The Layers of Earth in Term of Fractional Derivative: A Study," *Gomal University Journal of Research*, vol 34(2), pp. 1-13, 2018.
- [38] Saqlain M, Jafar N, Hamid R, Shahzad A. "Prediction of Cricket World Cup 2019 by TOPSIS Technique of MCDM-A Mathematical Analysis," *International Journal of Scientific & Engineering Research*, vol 10(2), pp. 789-792, 2019.
- [39] Saqlain M, Saeed M, Ahmad M. R, Smarandache, F. "Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application," *Neutrosophic Sets and Systems (NSS)*, vol 27, pp. 131-137, 2019.
- [40] Riaz.M., Saeed.M. Saqlain.M. and Jafar.N,"Impact of Water Hardness in Instinctive Laundry System based on Fuzzy Logic Controller," *Punjab University Journal of Mathematics*, vol 51(4), pp. 73-84, 2018.
- [41] Riaz. M., Saqlain. M. and Saeed. M., "Application of Generalized Fuzzy TOPSIS in Decision Making for Neutrosophic Soft set to Predict the Champion of FIFA 2018: A Mathematical Analysis," *Punjab University Journal of Mathematics*, vol 51(8), pp.111-126, 2019.
- [42] I. Deli and S. Broumi, "Neutrosophic Soft Matrices and NSM-decision Making," *Journal of Intelligent and Fuzzy Systems*, vol 28(5), pp. 2233-2241 2015.
- [43] T. Bera and N. K. Mahapatra, Introduction to neutrosophic soft groups, *Neutrosophic Sets and Systems*, vol 13, pp. 118-127, 2016. doi.org/10.5281/zenodo.570845.
- [44] P. Biswas, S. Pramanik, and B. C. Giri. "A new methodology for neutrosophic multi-attribute decision making with unknown weight information," *Neutrosophic Sets and Systems*, vol 3, pp. 42-52, 2014.

- [45] K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. *Neutrosophic Sets and Systems*, vol 7, pp. 62-68, 2015.
- [46] Smarandache, F., Pramanik, S., "New Neutrosophic Sets via Neutrosophic Topological Spaces," In *Neutrosophic Operational Research*; Eds.; Pons Editions: Brussels, Belgium, vol I, pp. 189–209, 2017.
- [47] Saqlain, M. Sana, M., Jafar. M. N., Saeed, M., Smarandache, F. "Aggregate Operators of Neutrosophic Hypersoft Set," *Neutrosophic Sets and Systems*, vol. 32, pp. 294-306, 2020. DOI: 10.5281/zenodo.3723155
- [48] Saqlain, M., Jafar, M. N., Riaz, M. "A New Approach of Neutrosophic Soft Set with Generalized Fuzzy TOPSIS in Application of Smart Phone Selection," *Neutrosophic Sets and Systems*, vol. 32, pp. 307-316, 2020. DOI: 10.5281/zenodo.3723161.
- [49] Saqlain, M., Jafar, M. N., Moin, S., Saeed, M. and Broumi, S. "Single and Multi-valued Neutrosophic Hypersoft set and Tangent Similarity Measure of Single valued Neutrosophic Hypersoft Sets," *Neutrosophic Sets and Systems*, vol. 32, pp. 317-329, 2020. DOI: 10.5281/zenodo.3723165
- [50] A. Chakraborty, S. Broumi, P.K Singh, "Some properties of Pentagonal Neutrosophic Numbers and its Applications in Transportation Problem Environment," *Neutrosophic Sets and Systems*, vol.28, pp.200- 215, 2019.
- [51] A. Chakraborty, S. Mondal, S. Broumi, "De-Neutrosophication technique of pentagonal neutrosophic number and application in minimal spanning tree," *Neutrosophic Sets and Systems*, vol. 29, pp. 1-18, 2019. doi: 10.5281/zenodo.3514383.
- [52] Edalatpanah, S. A., "A Direct Model for Triangular Neutrosophic Linear Programming," *International Journal of Neutrosophic Science*, Volume 1, Issue 1, pp. 19-28, 2020.
- [53] Chakraborty, A. "A New Score Function of Pentagonal Neutrosophic Number and its Application in Networking Problem," *International Journal of Neutrosophic Science*, Volume 1, Issue 1, pp. 40-51, 2020.
- [54] Parimala ,M., Karthika, M, Florentin Smarandache , Said Broumi, "On  $\alpha\omega$ -closed sets and its connectedness in terms of neutrosophic topological spaces," *International Journal of Neutrosophic Science*, Volume 2 , Issue 2, pp. 82-88, 2020.
- [55] Pratihari, J.; Kumar, R.; Dey, A.; Broumi, S. Transportation problem in neutrosophic environment. In *Neutrosophic Graph Theory and Algorithms*; IGI Global, pp.180-212, 2020.
- [56] Pratihari, J.; Kumar, R.; Edalatpanah, S. A.; Dey, A. Modified Vogel's approximation method for transportation problem under uncertain environment. *Complex & Intelligent Systems*, pp.1-12, 2020.
- [57] Mohapatra, H.; Panda, S.; Rath, A. K.; Edalatpanah, S. A.; Kumar, R. A tutorial on powershell pipeline and its loopholes. *International Journal of Emerging Trends in Engineering Research*, 8, pp.975-982,2020.
- [58] Kumar, R.; Edalatpanah, S. A.; Mohapatra, H. A note on "Optimal path selection approach for fuzzy reliable shortest path problem." *Journal of Intelligent & Fuzzy Systems*, pp.1-4, 2020.
- [59] Kumar, R.; Dey, A.; Broumi, S.; Smarandache, F. A study of neutrosophic shortest path problem. In *Neutrosophic Graph Theory and Algorithms*; IGI Global, pp.148-179, 2020.
- [60] Gayen, S.; Jha, S.; Singh, M.; Kumar, R. On a generalized notion of anti-fuzzy subgroup and some

characterizations. *International Journal of Engineering and Advanced Technology* 2019, 8, 385-390.

- [61] Gayen, S.; Smarandache, F.; Jha, S.; Singh, M. K.; Broumi, S.; Kumar, R. Introduction to plithogenic subgroup. In *Neutrosophic Graph Theory and Algorithms*; IGI Global, pp.213-259, 2020.
- [62] Gayen, S.; Smarandache, F.; Jha, S.; Singh, M. K.; Broumi, S.; Kumar, R. Introduction to plithogenic hypersoft subgroup. *Neutrosophic Sets and Systems*, 33, pp.208-233,2020.
- [63] Gayen, S.; Smarandache, F.; Jha, S.; Kumar, R. Interval-valued neutrosophic subgroup based on interval-valued triple t-norm. In *Neutrosophic sets in decision analysis and operations research*; IGI Global, pp 215-243,2020.



## An Expanded Model of Unmatter from Neutrosophic Logic perspective: Towards Matter-Spirit Unity View

Victor Christianto<sup>1</sup> Robert N. Boyd,<sup>2</sup> and Florentin Smarandache<sup>3</sup>

<sup>1</sup>Malang Institute of Agriculture (IPM), INDONESIA; [victorchristianto@gmail.com](mailto:victorchristianto@gmail.com)

<sup>2</sup>Consulting Physicist, Princeton Biotechnology Corporation, USA. Email: [rnboydphd@comcast.net](mailto:rnboydphd@comcast.net)

<sup>3</sup>Dept. Math and Sciences, University of New Mexico, Gallup, NM, USA; [smarand@unm.edu](mailto:smarand@unm.edu)

\* Correspondence: [victorchristianto@gmail.com](mailto:victorchristianto@gmail.com)

### Abstract

In Neutrosophic Logic, a basic assertion is that there are variations of about everything that we can measure; the variations surround three parameters called T, I, F (truth, indeterminacy, falsehood) which can take a range of values. A previous paper in IJNS, 2020 shortly reviews the links among aether and matter creation from the perspective of Neutrosophic Logic. In any case, matter creation process in nature stays a major puzzle for physicists, scientists and other science analysts. To this issue neutrosophic logic offers an answer: "unmatter." This paper examines an extended model of unmatter, to incorporate issue soul solidarity. So, neutrosophic logic may demonstrate helpful in offering goal to long standing clashes.

**Keywords:** Neutrosophic Logic, Physical Neutrosophy, aether, matter creation, unmatter, unparticle

### 1.Introduction

In accordance with the quick improvement of new part of basic science, for example neutrosophic logic, here we talk about possible use of NL hypothesis in the field of media transmission. See for ongoing papers: [31-35].

It is known that *matter creation* processes in nature remains a big mystery for physicists, biologists and other science researchers. To this problem neutrosophic logic offers a solution, i.e. *unmatter and unparticle*. See also previous papers on unmatter [21-27].

To put it plainly, neutrosophic logic may demonstrate helpful in offering goal to long standing clashes. See likewise our past papers on this issue. [1-2].

## 2. Matter creation processes and Grusenick experiment

Physicists all through numerous hundreds of years have bantered over the physical presence of aether medium. Since its origin by Isaac Newton, many accepted that it is required in light of the fact that in any case it is highly unlikely to clarify communication a good ways off in a vacuum space. We need mechanism of connection, of which has been called by different names, for example, quantum vacuum, zero point field, and so forth.

The celebrated Michelson-Morley tests were thought to give invalid outcome to aether speculation, and truly it was the premise of Einstein's STR. In any case, more up to date conversations demonstrated that the proof was fairly equivocal, from MM information itself. Particularly after Dayton Miller examinations of aether float were accounted for, an ever increasing number of information came to help aether speculation, albeit numerous physicists would lean toward another terms, for example, physical vacuum or superfluid vacuum. See [9-13].

In this regards, an experiment which is worthy to mention here is by Grusenick. Actually, his method is quite similar to Michelson-Morley experiment, except that he puts the interferometer *vertically*, which makes him able to detect the vertical aether inflow perpendicularly toward the surface of the Earth. Because only few papers discuss his result, let us give him space to tell in his own words, which can be paraphrased as follows:

"I have perused your information with much intrigue. Numerous individuals state that my development is precisely excessively flimsy, and that gravity impacts my contraption. So I assembled another. A man named Norbert Feist gave me better optical hardware to utilize. The new interferometer is just a steel plate with 189mm width and 8mm thick. The mirrors and the mirror holders are fabricated by Edmund, USA. Their shaft splitter anyway is precisely excessively insecure, so I utilized the one I made myself.

The obstruction design is anticipated on a little bit of paper. During a 180° pivot with the new Interferometer, I can see on normal 1.5 impedance periphery shifts during the night and 2.0 during daytime. With the more established one, which you can find in the YouTube film, there are 11.0 movements around evening time and 11.5 if the trial is performed during daytime. In this way, the two Interferometers (the more seasoned and the more up to date one) show a distinction of 0.5 obstruction periphery shifts among day and night.

I additionally might want to make reference to that a slight variety in the quality of the periphery design development happens during various days of the month. On Thursday 16.10.2009 at 24.00 o'clock, I could see a full 3.0 obstruction periphery shifts per 180° pivot (with the new interferometer). The zero point, where a stop of the example development occurs, is for the two interferometers at a similar position. There are two zero focuses in a 360° turn of the two interferometers when the shaft splitter is situated evenly to the world's surface. To all individuals who state that the main impact on the interferometer is gravity, I have a straightforward inquiry. Why would that be no zero point or stop of the periphery design development when the shaft splitter is in the vertical position? In the pillar splitter's vertical position, the mirrors and the mirror holders are evenly pushed by gravity. In any case, there is no zero point." [19]

According to Paul LaViolette, Grusenick's experiment proves the existence of ether and also his G-ons theory:

"Subquantum Kinetics requires that G etherons (G-ons) reliably diffuse into the Earth, driven by the incline in the Earth's 1/r gravitational conceivable field. The low G-on center inside the Earth, as differentiated and the Earth's space condition, develops considering the way that G-ons are foreseen to be conveyed at an all the more moderate rate in the unbiased issue inside the Earth as differentiated and enveloping space. ... Later

he built up an improved adjustment of the interferometer, showed as follows, and found a total fringe move of 1.5 to 2.0 as the mechanical gathering was turned in the vertical bearing. This value comes closer to that of U.S. investigator Frank Pearce who has played out a type of the Grusenick break down using a 1 inch thick stone square, as opposed to an aluminum board, for mounting the interferometer mirrors and who found a move of just around one half to one outskirts when the mechanical get together was turned in the vertical bearing.”[20]

Alternatively, let us assume that under certain conditions that aether can transform using Bose condensation process to become “*unmatter*”, a transition phase of material, which then it sublimates into matter (solid, gas, liquid). Unmatter can also be considered as “*pre-physical matter*.”

Summarizing our idea, it is depicted in the following block diagram [1]:

Aether → bose condensation → “unmatter” (pre-physical  
matter) → sublimation → ordinary matter/particle

Diagram 1. How aether becomes ordinary matter

Actually the term “unmatter” can be viewed as a solution from perspective of Neutrosophic Logic. A bit of history of unmatter term may be useful here:

*“The word 'Unmatter' was instituted by one of us (F. Smarandache) and distributed in 2004 of every three papers regarding the matter. Unmatter is framed by mixes of issue and antimatter that bound together, or by long-extend blend of issue and antimatter shaping a pitifully coupled stage. The possibility of unparticle was first considered by F. Smarandache in 2004, 2005 and 2006, when he transferred a paper on CERN site and he distributed three papers about what he called 'unmatter', which is another type of issue framed by issue and antimatter that quandary together. Unmatter was presented with regards to 'neutrosophy' (Smarandache, 1995) and 'paradoxism' (Smarandache, 1980), which depend on blends of inverse substances 'An' and 'antiA' along with their neutralities 'neutA' that are in the middle.”<sup>1</sup> See also Christiano & Smarandache [17]. See also F. Smarandache et al.’s papers and books, [21-27].*

In any case, in this paper, unmatter is considered as a progress state (pre-physical) from aether to get common particles, see also [1]. Moreover, superfluid model of dark matter has been discussed by some authors [6-7].

### 3. An expanded model of *unmatter*

In other side, it is known that astronomers find that only 1% of matter in the universe is observed, while 99% is undetected. That is why they call it the Hidden Universe.

<sup>1</sup> <http://fs.unm.edu/unmatter.htm>

Could it be that aether (may be in form of superfluid medium, a ka Mishin phase state) can be intermediate entity in neutrosophic sense?

In this line of thought, it is possible to come up with an expanded model of unmatter, as follows:

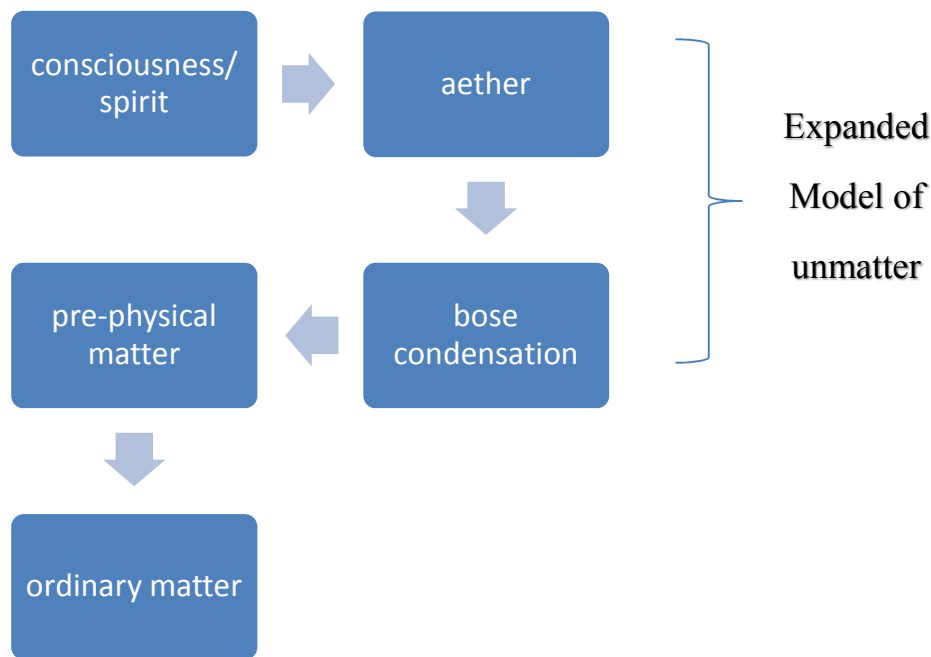


Diagram 2. An *expanded model of unmatter*

May be it is because the remaining entities are in the form of consciousness, aether and pre-physical matter. That is what can be called as “*expanded model of unmatter.*”

#### 4. Remark on grid cells, bhutatmas, and consciousness

May be it is possible to come up with a model of how spirit affect matter and vice versa, which reminds us to papers by Ervard Moser et al. on grid cells, space cells etc.

We can add some remark as follows:

“Space cells and grid cells were first discussed by Alfven (Nobel Prize in Physics) regarding plasma behaviors. I brought it out that these cells have, and evoke, personality traits in all who occupy the given cell, over large spans of time. Which means each star and each cell in the spaces between stars, has a unique personality.

The Russians did a research project that covered most of Asia, and all of Europe, and determined that each cell contains life forms plants animals birds insects fish, and so on, that are unique to that specific cell, and that the people who are native to a given cell have similar personality traits and behaviors unlike the inhabitants of other cells.

The personality of the land of a given cell produces an ambient "personality field" and a unique "magic" that can be learned and used by the inhabitants as a source of benefits which are specific to that cell.

These personality cells are produced by the aether energy-information contents of the plasmas which originate the personality of the given cell. These cell have distinct boundaries and are directly involved with creating life-forms which are perfectly suited to life in the given cell. Some life forms are able to cross over into other cells without undue stresses. Others do not live long when they are removed from their native information-energy habitat.

For life forms which are able to transit and occupy various cells, if the given Being spends a large amount of time in a specific cell, they start to change physically and psychologically in alignment with the qualities of the personality of the land they are spending large amounts of time in.

The Bhutatmas are conveyed by plasmas and "stick to" every material form. Aether clings to matter at all scales, interpenetrating it and forming an atmosphere, similar to the photographs taken by Krasnoholovets of the "atmosphere" of "inertons" which surround electrons. Inertons are much larger than Bhutatmas, however. There are many layers of behaviors related to the smallness of the entities involved which form thresholds of altered physical behaviors, as seen in Pendry structures and other metamaterials.

Air currents, water currents, electrical flows, plasma flows, and all wave forms in all media, regardless of phase state, convey aether and information energy between end points and all along the lines of the flows. Aether circuits are always bi-directional between end-points, while plasma and electrical flows are one-directional. Gravitation and time aether flows also carry information-energy and can alter the given local energetic informational environment fairly rapidly, or over large spans of time.

Marjanovic's model does not cover any of this, as he has no attention for the physics of information-energy, Consciousness, nor studies of the activities of Divinity. Bhutatmas cover all the bases."

Moreover we can add...

"Personality cells are determinant in what kinds of matter are formed, and in where and when they are formed. Stars each have a unique personality, a unique chemistry, and a unique radiation spectrum, exactly because they are formed in different cells with different personalities, which personality cells act as environmental factors during star formation and planet formation.

This is related to the Telluric Intelligence (inhabiting aether rivers) which is endemic to and inherent in each star and each planet. Probably, each Telluric Intelligence is unique, as well as being involved with the unique star and the unique planets associated with the given star.

According to Wal Thornhill and Steven Smith, with whom we agree on this, planets are formed internal to stars by precipitation processes resulting from the creation of atomic elements in the outer-most layers of stars, due to charge separation in stellar plasmas creating enormous gradients in the stellar electric field, thus urging the aether involved with the given star to create new atoms, as put into evidence in the SAFIRE Project. The newly formed atoms tend to precipitate and drift towards the central regions of the given star. Each planet will be unique, but have some traits in common with its parent star.

Uniqueness is partly due to non-local influences being imposed on stellar systems by infinite velocity infinitesimals which carry and convey information to the given stellar system which influence the personality and material composition of the given star. This is a process due to the 5th phase state in Mishin's 5 phase-state aether. (Tesla talks about non-local influences imparting information and various forms of organization to local systems.) This is in addition to the Personality information inherent to the given aether-plasma space-cell, which can modify the local personality over time, and in response to superluminal activities of quantized red-shifts resulting in local variations in the laws of physics in the region, and local variations in the fine structure "constant", leaving leaving the galaxy-core aether-plasmoid in superluminal 3D shells, modifying the physics in the volume of the given out-bound shell."

Hopefully many more approaches can be developed in the direction as mentioned above.

## 5. Concluding remarks

In this paper, we discussed three possible applications of Neutrosophic Logic in the field of matter creation processes etc. For instance, a redefinition of term "unmatter" is proposed here, where under certain conditions, aether can transform using Bose condensation process to become "*unmatter*", a transition phase of material, which then it sublimates into matter (solid, gas, liquid). Unmatter can also be considered as "pre-physical matter." Moreover, we can extend it further to include consciousness/spirit, which may explain why the 99% of matter in this Universe is undetected. Further researches are recommended in the above directions.

## Acknowledgment

The authors wish to thank to several anonymous reviewers for suggesting improvements to this manuscript.

**Funding:** “This research received no external funding”

**Conflicts of Interest:** “The authors declare no conflict of interest.”

## References

- [1] V. Christianto, R.N. Boyd & F. Smarandache. Three Possible Applications of Neutrosophic Logic in Fundamental and Applied Sciences. *IJNS* Vol. 1 no. 2, pp. 90-95, 2020.
- [2] V. Christianto, F. Smarandache, R.N. Boyd. Electron Model Based Helmholtz’s Electron Vortex & Kolmogorov’s Theory of Turbulence. *Prespacetime J.* Vol. 10 no. 1 (2019) url: <https://prespacetime.com/index.php/pst/article/view/1516>
- [3] V. Christianto & F. Smarandache. One-note-Samba approach to cosmology. *Prespacetime J.*, Vol. 10, no. 6 (2019). <https://prespacetime.com/index.php/pst/article/view/1586/1532>
- [4] <https://www.cosmic-core.org/free/article-99-the-aether-part-2-aether-advocates-experiments/>
- [5] Sinha, Sivaram, Sudharsan. Aether as a superfluid state of particle-antiparticle pairs. *Foundations of Physics*, Vol. 6, No.1, 1976.
- [6] Sinha. Bose condensation of particle-antiparticle systems. *Pramana – J. Phys.* Vol. 25, 1985.
- [7] Lasha Berezhiani and Justin Houry. Dark Matter Superfluidity and Galactic Dynamics. arXiv:1506.07877 (2015)
- [8] Martin J. Cooper. Vortex Motions in Ideal Bose Superfluid. *JOURNAL OF RESEARCH of the National Bureau of Standards - A. Physics and Chemistry* Vol. 73A, No.3, May-June 1969.
- [9] Swenson, Loyd S. "The Michelson–Morley–Miller Experiments before and after 1905," *Journal for the History of Astronomy*. 1 (2): 56–78, 1970.
- [10] Swenson, Loyd S. The Ethereal ether: A History of the Michelson-Morley-Miller Aether-drift Experiments, 1880-1930. The University of Texas – Austin, 2013. url: [https://books.google.co.id/books/about/The\\_Ethereal\\_Aether.html?id=kQTUAAAAQBAJ&redir\\_esc=y](https://books.google.co.id/books/about/The_Ethereal_Aether.html?id=kQTUAAAAQBAJ&redir_esc=y)
- [11] Dayton Miller. The Ether-Drift Experiment and the Determination of the Absolute Motion of the Earth", *Reviews of Modern Physics*, Vol.5(2), p.203-242, July 1933. See also [23a] Dayton Miller, untitled lecture in "Conference on the Michelson-Morley Experiment", *Astrophysical Journal*, LXVIII:341-402, Dec. 1928; also in *Contributions From the Mount Wilson Observatory*, No.373, Carnegie Institution of Washington
- [12] James DeMeo. Dayton J. Miller revisited. In *Should the Laws of Gravitation be Reconsidered?* Hector Monera, Editor, Alleiron, Montreal 2011, 11.285-315. url: <http://www.orgonelab.org/demeopubsPDFs/2011Miller.pdf>
- [13] M. Consoli & A. Pluchino. *Michelson-Morley Experiments: An Enigma for Physics and the History of Science*. World Scientific, 2018. url: <https://www.worldscientific.com/worldscibooks/10.1142/11209>
- [14] James Clark Maxwell. On Physical Line of Forces. [26a] Maxwell, J. C., *A Treatise on Electricity and Magnetism*, 3rd edition, Dover, New York, 1954; The Dover edition is an unabridged, slightly altered, republication of the third edition, published by the Clarendon Press in 1891.
- [15] Arthur D. Yaghjian. Reflections on Maxwell’s Treatise. *Progress In Electromagnetics Research*, Vol. 149, 217–249, 2014
- [16] Robert Kowalski. A logic-based approach to conflict resolution. url: <http://www.doc.ic.ac.uk/~rak/papers/conflictresolution.pdf>
- [17] V. Christianto & F. Smarandache. A Review of Seven Applications of Neutrosophic Logic: In Cultural Psychology, Economics Theorizing, Conflict Resolution, Philosophy of Science, etc. *J* 2019, 2(2), 128-137; <https://doi.org/10.3390/j2020010>
- [18] Lydia Patton. Reconsidering Experiments. *HOPOS: The Journal of the International Society for the History of Philosophy of Science*. Vol. 1, No. 2. 2011. Pages 209-226. Official URL: <http://www.journals.uchicago.edu/doi/abs/10.1086/660167>

- [19] Martin Grusenick's repetition of the Michelson-Morley interference experiment. url: <http://worldnpa.ning.com/video/extended-michelsonmorley>
- [20] Paul LaViolette. Grusenick experiment proves the existence of the ether and vertical inflow of ether. url: <https://etheric.com/grusenick-experiment-proves-existence-ether/>
- [21] F. Smarandache. Matter, Antimatter, and Unmatter. *Prog. Phys.* 1, 2005. Preprint: EXT-2004-142. url: <http://cdsweb.cern.ch/record/798551>
- [22] F. Smarandache. Verifying Unmatter by Experiments, More Types of Unmatter. *Prog. Phys.* 2, 2005. url: <http://www.ptep-online.com>
- [23] F. Smarandache & D. Rabounski. Unmatter Entities inside Nuclei, Predicted by the Brightsen Nucleon Cluster Model, *Progress in Physics*, 1/2006. url: <http://www.ptep-online.com>
- [24] E. Goldfain & F. Smarandache. On Emergent Physics, "Unparticles" and Exotic "Unmatter" States, *Prog. Phys.* Vol. 4, 10-15, 2008. url: <http://www.ptep-online.com>
- [25] F. Smarandache. Unmatter Plasma Discovered. *Prog. Phys.* Vol. 11, 2015. url: <http://www.ptep-online.com>
- [26] F. Smarandache. *Unmatter Plasma, Relativistic Oblique-Length Contraction Factor, Neutrosophic Diagram and Neutrosophic Degree of Paradoxicity. Articles and Notes.* Brussels: Pons Asbl, 2015. ISBN 978-1-59973-346-3
- [27] F. Smarandache. *Proceedings of the Introduction to Neutrosophic Physics: Unmatter & Unparticle International Conference.* December 2-4, 2011. Ohio: Zip Publ, 2011. ISBN: 9781599731827,
- [28] G. Marjanovic. Pyramids - guardians of primary cosmic vibration of the local densities. *Second International Scientific Conference on Bosnian Valley of the Pyramids*, September 5-10, 2011 Visoko, Bosnia-Herzegovina.
- [29] R.N Boyd. Presentation at *Physics Beyond Relativity Conference*, Prague, Nov. 2019. Link to presentation: <https://science21.cz/conference/?p=901>
- [30] Wallace Thornhill on planet formation: "Wal Thornhill: The Saturn/Earth Connection and Our Place in the Universe | Space News". Link to video presentations: url: <https://youtu.be/6fjcPguafug>, url: <https://www.youtube.com/watch?v=UkPpR8t1qvk>. See also on local brown dwarf star (proto-Saturn): <https://youtu.be/6fjcPguafug?t=420>
- [31] Madeleine Al- Tahan, Some Results on Single Valued Neutrosophic (Weak) Polygroups, *International Journal of Neutrosophic Science*, Volume 2 , Issue 1, pp. 38-46, 2020.
- [32] Mohsin Khalid, Young Bae Jun, Mohammad Mohseni Takallo, Neha Andaleeb Khalid, Magnification of MBJ-Neutrosophic Translation on G-Algebra, *International Journal of Neutrosophic Science* Volume 2 , Issue 1, pp. 27-37, 2020
- [33] S. A. Edalatpanah, A Direct Model for Triangular Neutrosophic Linear Programming, *International Journal of Neutrosophic Science*, Volume 1, Issue 1, pp. 19-28, 2020
- [34] M. Parimala , M. Karthika , Florentin Smarandache , Said Broumi, On  $\alpha\omega$ -closed sets and its connectedness in terms of neutrosophic topological spaces, *International Journal of Neutrosophic Science*, Volume 2 , Issue 2, pp. 82-88, 2020
- [35] E.O. Adeleke , A.A.A. Agboola , F. Smarandache, Refined Neutrosophic Rings II, *International Journal of Neutrosophic Science*, Volume 2 , Issue 2, pp. 89-94, 2020



## A Note on Single Valued Neutrosophic Sets in Ordered Groupoids

<sup>1</sup>M. Al-Tahan, <sup>2</sup>B. Davvaz, <sup>3</sup>M. Parimala

<sup>1</sup>Department of Mathematics, Lebanese International University, Bekaa, Lebanon

<sup>2</sup>Department of Mathematics, Yazd University, Yazd, Iran

<sup>3</sup>Department of Mathematics, Bannari Amman Institute of Technology, India

madeline.tahan@liu.edu.lb<sup>1</sup>, davvaz@yazd.ac.ir<sup>2</sup>, rishwanthpari@gmail.com<sup>3</sup>

### Abstract

The aim of this paper is to combine the notions of ordered algebraic structures and neutrosophy. In this regard, we define for the first time single valued neutrosophic sets in ordered groupoids. More precisely, we study single valued neutrosophic subgroupoids of ordered groupoids, single valued neutrosophic ideals of ordered groupoids, and single valued neutrosophic filters of ordered groupoids. Finally, we present some remarks on single valued neutrosophic subgroups (ideals) of ordered groups.

**Keywords:** SVNS,  $(\alpha, \beta, \gamma)$ -level set, ordered groupoid, single valued neutrosophic subgroupoid, single valued neutrosophic ideal, single valued neutrosophic filter.

## 1 Introduction

Neutrosophy [10], a new branch of philosophy that deals with indeterminacy, was launched by Smarandache in 1998. The theory of neutrosophy is based on the concept of indeterminacy (neutrality) that is neither true nor false. Smarandache [11] defined neutrosophic sets as a generalization of the fuzzy sets introduced by Zadeh [15] in 1965 and as a generalization of intuitionistic fuzzy sets introduced by Atanassov [4] in 1986. A special type of neutrosophic set is *single valued neutrosophic set (SVNS)* [14] which also can be considered as a generalization of fuzzy sets and intuitionistic fuzzy sets. In an SVNS, each element has a truth value “ $t$ ”, indeterminacy value “ $i$ ”, and a falsity value “ $f$ ” where  $0 \leq t, i, f \leq 1$  and  $0 \leq t + i + f \leq 3$ . When  $i = 0$  and  $f = 1 - t$ , we get a fuzzy set and when  $0 \leq t + f \leq 1$  and  $i = 1 - t - f$ , we get an intuitionistic fuzzy set. Neutrosophic sets have many applications in different fields of Science and Engineering. In particular, they are connected to various fields of Mathematics and especially to Algebra. For example, many researchers [1, 2, 9, 12, 13] have worked on the connection between neutrosophy and algebraic structures.

Our paper introduces a new link between algebraic structures and neutrosophy. In particular, it is concerned about single valued neutrosophic sets in ordered groupoids and it is organized as follows: after an Introduction, in Section 2, we present some definitions related to neutrosophy that are used throughout the paper. In Section 3, we present some definitions about ordered groupoids (groups) and elaborate some examples that are used in Section 4 and Section 5. In Section 4, we define single valued neutrosophic subgroupoids (ideals) as well as single valued neutrosophic filters of ordered groupoids, present many non-trivial examples about the new defined concepts, and study some of their properties. Finally in Section 5, we apply the definition of SVNS in ordered groupoids to ordered groups and present some remarks and results.

## 2 Single valued neutrosophic sets

In this section, we present some definitions about neutrosophy that are used throughout the paper.

**Definition 2.1.** [14] Let  $X$  be a non-empty space of elements (objects). A single valued neutrosophic set (SVNS)  $A$  on  $X$  is characterized by truth-membership function  $T_A$ , indeterminacy-membership function  $I_A$ , and falsity-membership function  $F_A$ . For each element  $x \in X$ ,  $0 \leq T_A(x), I_A(x), F_A(x) \leq 1$ .

**Definition 2.2.** [3] Let  $X$  be a non-empty set,  $0 \leq \alpha, \beta, \gamma \leq 1$ , and  $A$  an SVN over  $X$ . Then the  $(\alpha, \beta, \gamma)$ -level set of  $A$  is defined as follows:

$$L_{(\alpha, \beta, \gamma)} = \{x \in X : T_A(x) \geq \alpha, I_A(x) \leq \beta, F_A(x) \leq \gamma\}.$$

**Definition 2.3.** [14] Let  $X$  be a non-empty set and  $A, B$  be single valued neutrosophic sets over  $X$  defined as follows.

$$A = \left\{ \frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in X \right\}, B = \left\{ \frac{x}{(T_B(x), I_B(x), F_B(x))} : x \in X \right\}.$$

Then

1.  $A$  is called a single valued neutrosophic subset of  $B$  and denoted as  $A \subseteq B$  if  $T_A(x) \leq T_B(x)$ ,  $I_A(x) \leq I_B(x)$ , and  $F_A(x) \geq F_B(x)$  for all  $x \in X$ .

If  $A$  is a single valued neutrosophic subset of  $B$  and  $B$  is a single valued neutrosophic subset of  $A$  then  $A$  and  $B$  are said to be equal single valued neutrosophic sets ( $A = B$ ).

2. The union of  $A$  and  $B$  is defined to be the SVN over  $X$ :

$$A \cup B = \left\{ \frac{x}{(T_{A \cup B}(x), I_{A \cup B}(x), F_{A \cup B}(x))} : x \in X \right\}.$$

Here,  $T_{A \cup B}(x) = T_A(x) \vee T_B(x)$ ,  $I_{A \cup B}(x) = I_A(x) \vee I_B(x)$ , and  $F_{A \cup B}(x) = F_A(x) \wedge F_B(x)$  for all  $x \in X$ .

3. The intersection of  $A$  and  $B$  is defined to be the SVN over  $X$ :

$$A \cap B = \left\{ \frac{x}{(T_{A \cap B}(x), I_{A \cap B}(x), F_{A \cap B}(x))} : x \in X \right\}.$$

Here,  $T_{A \cap B}(x) = T_A(x) \wedge T_B(x)$ ,  $I_{A \cap B}(x) = I_A(x) \wedge I_B(x)$ , and  $F_{A \cap B}(x) = F_A(x) \vee F_B(x)$  for all  $x \in X$ .

**Example 2.4.** Let  $X = \{s, a, m\}$  and  $S, M$  be SVN over  $X$  defined as follows.

$$S = \left\{ \frac{s}{(0.7, 0.6, 0.5)}, \frac{a}{(0.8, 0.4, 0.2)}, \frac{m}{(0.1, 0.6, 1)} \right\},$$

$$M = \left\{ \frac{s}{(0.9, 0.1, 0.7)}, \frac{a}{(1, 0, 0.6)}, \frac{m}{(0.9, 0.3, 0.2)} \right\}.$$

Then the SVN  $S \cap M$  and  $S \cup M$  over  $X$  are as follows.

$$S \cap M = \left\{ \frac{s}{(0.7, 0.1, 0.7)}, \frac{a}{(0.8, 0, 0.6)}, \frac{m}{(0.1, 0.3, 1)} \right\},$$

$$S \cup M = \left\{ \frac{s}{(0.9, 0.6, 0.5)}, \frac{a}{(1, 0.4, 0.2)}, \frac{m}{(0.9, 0.6, 0.2)} \right\}.$$

### 3 Ordered groupoids and ordered groups

In this section, we present some examples on ordered groupoids and ordered groups that are used in Section 4 and Section 5. For more details about ordered algebraic structures, we refer to [5] and [6].

**Definition 3.1.** [5] Let  $(G, \cdot)$  be a groupoid (group) and " $\leq$ " be a partial order relation (reflexive, antisymmetric, and transitive) on  $G$ . Then  $(G, \cdot, \leq)$  is an ordered groupoid (ordered group) if the following condition holds for all  $z \in G$ .

$$\text{If } x \leq y \text{ then } z \cdot x \leq z \cdot y \text{ and } x \cdot z \leq y \cdot z.$$

**Definition 3.2.** Let  $(G, \cdot, \leq)$  be an ordered groupoid (group). Then  $G$  is called a *total ordered groupoid (group)* if  $x$  and  $y$  are comparable for all  $x, y \in G$ . i.e.,  $x \leq y$  or  $y \leq x$  for all  $x, y \in G$ .

An ordered groupoid  $(G, \cdot, \leq)$  is said to be *commutative* if  $x \cdot y = y \cdot x$  for all  $x, y \in G$  and an element  $e$  in an ordered groupoid  $(G, \cdot, \leq)$  is called an *identity* if  $e \cdot x = x \cdot e = x$  for all  $x \in G$ . If such an element exists then it is unique.

**Remark 3.3.** Let  $(G, \cdot)$  be any groupoid (group). Then by defining “ $\leq$ ” on  $G$  as follows: For all  $x, y \in G$ ,

$$x \leq y \iff x = y.$$

Then  $(G, \cdot, \leq)$  is an ordered groupoid (group).

Such an order is called the **trivial order**.

Ordered groups are a special case of ordered groupoids. We present some examples on infinite ordered groups.

**Example 3.4.** The groups of integers, rational numbers, real numbers under standard addition and usual order are ordered groups.

**Example 3.5.** Let  $\mathbb{Q}^+$  be the set of positive rational numbers. Then  $(\mathbb{Q}^+, \cdot, \leq)$  is an ordered group. Where “ $\leq$ ” is defined as follows: For all  $q, q' \in \mathbb{Q}^+$ ,

$$q \leq q' \iff \frac{q'}{q} \in \mathbb{N}.$$

We show that the partial order “ $\leq$ ” defines an order on  $\mathbb{Q}^+$ . Let  $q \leq q'$  and  $z \in \mathbb{Q}^+$ . Having  $\frac{q'}{q} \in \mathbb{N}$  and  $z > 0$  implies that  $\frac{q'z}{qz} \in \mathbb{N}$ . Thus,  $qz \leq q'z$ .

As an illustration for “ $\leq$ ” on  $\mathbb{Q}^+$ , we can say that  $\frac{1}{4} \leq \frac{1}{2}$  as  $\frac{\frac{1}{2}}{\frac{1}{4}} = 2 \in \mathbb{N}$  whereas,  $\frac{1}{4} \not\leq \frac{1}{3}$  as  $\frac{\frac{1}{3}}{\frac{1}{4}} = \frac{4}{3} \notin \mathbb{N}$ .

We present some examples on ordered groupoids that are not ordered groups.

**Example 3.6.** Let  $G$  be any non-empty set with  $a \in G$  and “ $\leq$ ” a partial order on  $G$ . Then by setting  $x \cdot y = a$  for all  $x, y \in G$ , we get that  $(G, \cdot, \leq)$  is an ordered groupoid.

**Example 3.7.** Let  $\mathbb{N} = \{1, 2, \dots\}$  be the set of natural numbers and define “ $\leq_{\mathbb{N}}$ ” in  $\mathbb{N}$  as follows: For all  $x, y \in \mathbb{N}$ ,

$$x \leq_{\mathbb{N}} y \text{ if and only if } x \geq y.$$

Then  $(\mathbb{N}, +, \leq_{\mathbb{N}})$  is a commutative ordered groupoid. This is easily seen as  $\leq_{\mathbb{N}}$  is a partial order on  $\mathbb{N}$  and if  $x \leq_{\mathbb{N}} y$  and  $z \in \mathbb{N}$  then  $x + z \geq y + z$  and hence  $x + z \leq_{\mathbb{N}} y + z$ .

Finite groupoids can be presented by means of Cayley’s table.

**Example 3.8.** Let  $(G_1, \cdot_1)$  be the groupoid defined by Table 1

Table 1: The groupoid  $(G_1, \cdot_1)$

$\cdot_1$	a	b	c
a	a	a	a
b	a	a	c
c	a	a	a

By setting  $\leq_1 = \{(a, a), (a, b), (a, c), (b, b), (c, c)\}$ , we get that  $(G_1, \cdot_1, \leq_1)$  is a commutative ordered groupoid.

**Example 3.9.** Let  $(G_1, \star)$  be the groupoid defined by Table 2

Table 2: The groupoid  $(G_1, \star)$

$\star$	a	b	c
a	a	a	a
b	a	a	c
c	a	c	a

By setting  $\leq_1 = \{(a, a), (a, b), (a, c), (b, b), (c, c)\}$ , we get that  $(G_1, \star, \leq_1)$  is an ordered groupoid.

**Example 3.10.** Let  $(G_2, \cdot_2)$  be the groupoid defined by Table 3

By setting  $\leq_2 = \{(1, 1), (2, 2), (2, 3), (3, 3), (4, 4)\}$ , we get that  $(G_2, \cdot_2, \leq_2)$  is an ordered groupoid that is not a total ordered groupoid.

Table 3: The groupoid  $(G_2, \cdot_2)$ 

$\cdot_2$	1	2	3	4
1	4	4	4	4
2	4	4	4	4
3	4	4	4	4
4	4	4	4	1

Table 4: The groupoid  $(G_3, \cdot_3)$ 

$\cdot_3$	e	c	d
e	e	c	d
c	c	c	c
d	d	c	d

Table 5: The groupoid  $(G_4, \cdot_4)$ 

$\cdot_4$	1	2	3
1	1	1	1
2	1	1	1
3	1	1	3

**Example 3.11.** Let  $(G_3, \cdot_3)$  be the groupoid defined by Table 4.

By setting  $\leq_3 = \{(e, e), (c, e), (c, d), (d, e), (d, d)\}$ , we get that  $(G_3, \cdot_3, \leq_3)$  is a total ordered groupoid with an identity “e”.

**Example 3.12.** Let  $(G_4, \cdot_4)$  be the groupoid defined by Table 5.

By setting  $\leq_4 = \{(1, 1), (1, 3), (2, 2), (2, 1), (2, 3), (3, 3)\}$ , we get that  $(G_4, \cdot_4, \leq_4)$  is a commutative total ordered groupoid.

**Definition 3.13.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $S \subseteq G$ . Then

$$(S] = \{x \in G : x \leq s \text{ for some } s \in S\}.$$

**Remark 3.14.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $S \subseteq G$ . Then  $S \subseteq (S]$ .

**Definition 3.15.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $S \subseteq G$ . Then

1.  $S$  is a *subgroupoid* of  $G$  if  $(S, \cdot)$  is a groupoid and  $(S] \subseteq S$ .
2.  $S$  is a *left ideal* of  $G$  if  $G \cdot S \subseteq S$  and  $(S] \subseteq S$ .
3.  $S$  is a *right ideal* of  $G$  if  $S \cdot G \subseteq S$  and  $(S] \subseteq S$ .
4.  $S$  is an *ideal* of  $G$  if it is a left ideal of  $G$  and a right ideal of  $G$ .

**Example 3.16.** In Example 3.10,  $\{1, 4\}$ ,  $\{1, 2, 4\}$ , and  $\{1, 2, 3, 4\}$  are ideals of  $(G_2, \cdot_2, \leq_2)$ .

**Definition 3.17.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $F \subseteq G$ . Then  $F$  is a *filter* of  $G$  if the following conditions are satisfied.

- (1)  $x \cdot y \in F$  for all  $x, y \in F$ ;
- (2) If  $x \cdot y \in F$  then  $x, y \in F$  for all  $x, y \in G$ ;
- (3) If  $x \in F, y \in G$  and  $x \leq y$  then  $y \in F$ .

**Example 3.18.** In Example 3.12,  $\{3\}$  and  $G_4$  are the only filters of  $(G_4, \cdot_4, \leq_4)$ .

## 4 SVN S in ordered groupoids

In this section and inspired by the definition of fuzzy sets in ordered groupoids [7], we define for the first time single valued neutrosophic subgroupoids (ideals) (in Subsection 4.1) as well as single valued neutrosophic filters (in Subsection 4.2) of ordered groupoids and study some of their properties such as finding a relationship between subgroupoids/ideals/filters of ordered groupoids and single valued neutrosophic subgroupoids/ideals/filters of these ordered groupoids. Moreover, we construct many non-trivial examples on them.

#### 4.1 Single valued neutrosophic subgroupoids (ideals) of groupoids

**Definition 4.1.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $G$ . Then  $A$  is single valued neutrosophic subgroupoid of  $G$  if for all  $x, y \in G$ , the following conditions hold:

- (1)  $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y)$ ;
- (2)  $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y)$ ;
- (3)  $F_A(x \cdot y) \leq F_A(x) \vee F_A(y)$ ;
- (4) If  $x \leq y$  then  $T_A(x) \geq T_A(y)$ ,  $I_A(x) \geq I_A(y)$ , and  $F_A(x) \leq F_A(y)$ .

**Definition 4.2.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $G$ . Then  $A$  is single valued neutrosophic left ideal of  $G$  if for all  $x, y \in G$ , the following conditions hold:

- (1)  $T_A(x \cdot y) \geq T_A(y)$ ;
- (2)  $I_A(x \cdot y) \geq I_A(y)$ ;
- (3)  $F_A(x \cdot y) \leq F_A(y)$ ;
- (4) If  $x \leq y$  then  $T_A(x) \geq T_A(y)$ ,  $I_A(x) \geq I_A(y)$ , and  $F_A(x) \leq F_A(y)$ .

**Definition 4.3.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $G$ . Then  $A$  is single valued neutrosophic right ideal of  $G$  if for all  $x, y \in G$ , the following conditions hold:

- (1)  $T_A(x \cdot y) \geq T_A(x)$ ;
- (2)  $I_A(x \cdot y) \geq I_A(x)$ ;
- (3)  $F_A(x \cdot y) \leq F_A(x)$ ;
- (4) If  $x \leq y$  then  $T_A(x) \geq T_A(y)$ ,  $I_A(x) \geq I_A(y)$ , and  $F_A(x) \leq F_A(y)$ .

**Definition 4.4.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $G$ . Then  $A$  is a single valued neutrosophic ideal of  $G$  if it is both: a single valued neutrosophic left ideal of  $G$  and a single valued neutrosophic right ideal of  $G$ .

**Remark 4.5.** Let  $(G, \cdot, \leq)$  be a commutative ordered groupoid and  $A$  an SVN over  $G$ . If  $A$  is a single valued neutrosophic right (or left) ideal of  $G$  then  $A$  is a single valued neutrosophic ideal of  $G$ .

**Remark 4.6.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $\alpha, \beta, \gamma \in [0, 1]$  be fixed values. Then

$$A = \left\{ \frac{x}{(\alpha, \beta, \gamma)} : x \in G \right\}$$

is single valued neutrosophic ideal of  $G$ . Moreover, it is called the **trivial single valued neutrosophic ideal**.

**Example 4.7.** Let  $(\mathbb{N}, +, \leq_{\mathbb{N}})$  be the ordered groupoid defined in Example 3.7 and  $A$  be an SVN over  $\mathbb{N}$  defined as follows: For all  $n \in \mathbb{N}$ ,

$$N_A(n) = \left( 1 - \frac{1}{n}, 1 - \frac{1}{n}, \frac{1}{n} \right).$$

Then  $A$  is a single valued neutrosophic ideal of  $\mathbb{N}$ . To prove that and by means of Remark 4.5, it suffices to show that  $A$  is a single valued neutrosophic right ideal of  $G$ . Let  $n, n' \in \mathbb{N}$ . Then  $n + n' \geq n$  and thus,  $T_A(n + n') = I_A(n + n') = 1 - \frac{1}{n+n'} \geq 1 - \frac{1}{n} = T_A(n) = I_A(n)$  and  $F_A(n + n') = \frac{1}{n+n'} \leq \frac{1}{n} = F_A(n)$ . Let  $n \leq_{\mathbb{N}} n'$ . Then  $n \geq n'$  and hence,  $T_A(n) = I_A(n) = 1 - \frac{1}{n} \geq 1 - \frac{1}{n'} = T_A(n') = I_A(n')$  and  $F_A(n) = \frac{1}{n} \leq \frac{1}{n'} = F_A(n')$ .

**Proposition 4.8.** Let  $(G, \cdot, \leq)$  be an ordered groupoid with identity  $e$  and  $A$  an SVN over  $G$ . Then  $A$  is a single valued neutrosophic left (right) ideal of  $G$  if and only if  $A$  is the trivial single valued neutrosophic ideal of  $G$ .

*Proof.* If  $A$  is the trivial single valued neutrosophic ideal of  $G$  then we are done by Remark 4.6. Conversely, let  $A$  be a single valued neutrosophic left (right) ideal of  $G$ . We prove the case when  $A$  is a single valued neutrosophic right ideal of  $G$  and the case when  $A$  is a single valued neutrosophic left ideal of  $G$  is done similarly. Let  $A$  be a single valued neutrosophic right ideal of  $G$ . Then for all  $x \in G$ , we have:

$$T_A(x) = T_A(e \cdot x) \geq T_A(e), I_A(x) = I_A(e \cdot x) \geq I_A(e), \text{ and } F_A(x) = F_A(e \cdot x) \leq F_A(e);$$

$$T_A(x) = T_A(x \cdot e) \geq T_A(x), I_A(x) = I_A(x \cdot e) \geq I_A(x), \text{ and } F_A(x) = F_A(x \cdot e) \leq F_A(x).$$

The latter implies that

$$T_A(x) = T_A(e), I_A(x) = I_A(e), \text{ and } F_A(x) = F_A(e).$$

Therefore,  $A$  is the trivial single valued neutrosophic ideal of  $G$ .  $\square$

**Example 4.9.** Proposition 4.8 asserts that the ordered groupoid  $(G_3, \cdot_3, \leq_3)$  in Example 3.11 has no non-trivial left (right) single valued neutrosophic ideals.

We present an example on a single valued neutrosophic right ideal that is not a single valued neutrosophic left ideal and an example on a single valued neutrosophic subgroupoid that is neither a single valued neutrosophic left ideal nor a single valued neutrosophic right ideal.

**Example 4.10.** Let  $(G_1, \cdot_1, \leq_1)$  be the ordered groupoid defined in Example 3.8 and  $A, B$  be the SVNS on  $G$  defined by  $N_A, N_B$  respectively as follows.

$$N_A(a) = (0.9, 0.8, 0.1), N_A(b) = N_A(c) = (0.7, 0.6, 0.2);$$

$$N_B(a) = (0.9, 0.8, 0.1), N_B(b) = (0.8, 0.5, 0.4), N_B(c) = (0.7, 0.6, 0.2).$$

Then  $A$  is a single valued neutrosophic ideal of  $G_1$  and  $B$  is a single valued neutrosophic right ideal of  $G_1$ . Moreover,  $B$  is not a single valued neutrosophic left ideal of  $G_1$  as  $T_B(b \cdot_1 c) = T_B(c) \not\geq T_B(b)$ .

**Example 4.11.** Let  $(G_1, \star, \leq_1)$  be the ordered groupoid defined in Example 3.9 and  $B$  be the SVNS on  $G$  defined by  $N_B$  as follows.

$$N_B(a) = (0.9, 0.8, 0.1), N_B(b) = (0.8, 0.5, 0.4), N_B(c) = (0.7, 0.6, 0.2).$$

Then  $B$  is a single valued neutrosophic subgroupoid of  $G_1$  that is neither a single valued neutrosophic left ideal of  $G_1$  nor a single valued neutrosophic right ideal of  $G_1$  as  $T_B(b \star c) = T_B(c \star b) = T_B(c) \not\geq T_B(b)$ .

**Lemma 4.12.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic subgroupoid of  $G$ . Then  $\bigcap_\alpha A_\alpha$  is a single valued neutrosophic subgroupoid of  $G$ .

*Proof.* Let  $x, y \in G$ . Then  $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(x) \wedge T_{A_\alpha}(y)$ ,  $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(x) \wedge I_{A_\alpha}(y)$ , and  $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(x) \vee F_{A_\alpha}(y)$  for all  $\alpha$ . The latter implies that

$$T_{\bigcap_\alpha A_\alpha}(x \cdot y) = \inf_\alpha T_{A_\alpha}(x \cdot y) \geq \inf_\alpha \{T_{A_\alpha}(x) \wedge T_{A_\alpha}(y)\} = \inf_\alpha T_{A_\alpha}(x) \wedge \inf_\alpha T_{A_\alpha}(y) = T_{\bigcap_\alpha A_\alpha}(x) \wedge T_{\bigcap_\alpha A_\alpha}(y);$$

$$I_{\bigcap_\alpha A_\alpha}(x \cdot y) = \inf_\alpha I_{A_\alpha}(x \cdot y) \geq \inf_\alpha \{I_{A_\alpha}(x) \wedge I_{A_\alpha}(y)\} = \inf_\alpha I_{A_\alpha}(x) \wedge \inf_\alpha I_{A_\alpha}(y) = I_{\bigcap_\alpha A_\alpha}(x) \wedge I_{\bigcap_\alpha A_\alpha}(y);$$

$$F_{\bigcap_\alpha A_\alpha}(x \cdot y) = \sup_\alpha F_{A_\alpha}(x \cdot y) \leq \sup_\alpha \{F_{A_\alpha}(x) \vee F_{A_\alpha}(y)\} = \sup_\alpha F_{A_\alpha}(x) \vee \sup_\alpha F_{A_\alpha}(y) = F_{\bigcap_\alpha A_\alpha}(x) \vee F_{\bigcap_\alpha A_\alpha}(y).$$

Let  $y \leq x$ . Then  $T_{A_\alpha}(y) \geq T_{A_\alpha}(x)$ ,  $I_{A_\alpha}(y) \geq I_{A_\alpha}(x)$ , and  $F_{A_\alpha}(y) \leq F_{A_\alpha}(x)$  for all  $\alpha$ . One can easily see that  $T_{\bigcap_\alpha A_\alpha}(y) \geq T_{\bigcap_\alpha A_\alpha}(x)$ ,  $I_{\bigcap_\alpha A_\alpha}(y) \geq I_{\bigcap_\alpha A_\alpha}(x)$ , and  $F_{\bigcap_\alpha A_\alpha}(y) \leq F_{\bigcap_\alpha A_\alpha}(x)$ . Therefore,  $\bigcap_\alpha A_\alpha$  is a single valued neutrosophic subgroupoid of  $G$ .  $\square$

**Remark 4.13.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic subgroupoid of  $G$ . Then  $\bigcup_\alpha A_\alpha$  may not be a single valued neutrosophic subgroupoid of  $G$ .

We illustrate Remark 4.13 by the following example.

**Example 4.14.** Let  $(\mathbb{N}, +, \leq)$  be the ordered groupoid of natural numbers under standard addition and trivial order. Define the SVNS  $A, B$  on  $\mathbb{N}$  as follows.

$$N_A(x) = \begin{cases} (0.9, 0.3, 0) & \text{if } x \text{ is a multiple of 2;} \\ (0, 0, 1) & \text{otherwise.} \end{cases}$$

$$N_B(x) = \begin{cases} (0.9, 0.3, 0) & \text{if } x \text{ is a multiple of 3;} \\ (0, 0, 1) & \text{otherwise.} \end{cases}$$

It is clear that  $A$  and  $B$  are single valued neutrosophic subgroupoids of  $\mathbb{N}$ . But  $A \cup B$  is not a single valued neutrosophic subgroupoid of  $\mathbb{N}$  as  $N_{A \cup B}(2 + 3) = N_{A \cup B}(5) = (0, 0, 1)$  so  $T_{A \cup B}(2 + 3) = T_{A \cup B}(5) = 0 \not\geq 0.9 = T_{A \cup B}(2) \wedge T_{A \cup B}(3)$ .

**Lemma 4.15.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic left (right) ideal of  $G$ . Then  $\bigcap_\alpha A_\alpha$  is a single valued neutrosophic left (right) ideal of  $G$ .

*Proof.* The proof is similar to that of Lemma 4.12.  $\square$

**Lemma 4.16.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic ideal of  $G$ . Then  $\bigcap_\alpha A_\alpha$  is a single valued neutrosophic ideal of  $G$ .

*Proof.* The proof follows from Lemma 4.15 and having an ideal of an ordered groupoid is a left ideal and right ideal of it.  $\square$

**Lemma 4.17.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic ideal of  $G$ . Then  $\bigcup_\alpha A_\alpha$  is a single valued neutrosophic ideal of  $G$ .

*Proof.* Let  $x, y \in G$ . Having  $A_\alpha$  a single valued neutrosophic right ideal of  $G$  implies that  $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(x)$ ,  $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(x)$ , and  $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(x)$  for all  $\alpha$ . The latter implies that

$$T_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha T_{A_\alpha}(x \cdot y) \geq \sup_\alpha T_{A_\alpha}(x) = T_{\bigcup_\alpha A_\alpha}(x);$$

$$I_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha I_{A_\alpha}(x \cdot y) \geq \sup_\alpha I_{A_\alpha}(x) = I_{\bigcup_\alpha A_\alpha}(x);$$

$$F_{\bigcup_\alpha A_\alpha}(x \cdot y) = \inf_\alpha F_{A_\alpha}(x \cdot y) \leq \inf_\alpha F_{A_\alpha}(x) = F_{\bigcup_\alpha A_\alpha}(x).$$

Similarly, having  $A_\alpha$  a single valued neutrosophic left ideal of  $G$  implies that  $T_{A_\alpha}(x \cdot y) \geq T_{A_\alpha}(y)$ ,  $I_{A_\alpha}(x \cdot y) \geq I_{A_\alpha}(y)$ , and  $F_{A_\alpha}(x \cdot y) \leq F_{A_\alpha}(y)$  for all  $\alpha$ . The latter implies that

$$T_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha T_{A_\alpha}(x \cdot y) \geq \sup_\alpha T_{A_\alpha}(y) = T_{\bigcup_\alpha A_\alpha}(y);$$

$$I_{\bigcup_\alpha A_\alpha}(x \cdot y) = \sup_\alpha I_{A_\alpha}(x \cdot y) \geq \sup_\alpha I_{A_\alpha}(y) = I_{\bigcup_\alpha A_\alpha}(y);$$

$$F_{\bigcup_\alpha A_\alpha}(x \cdot y) = \inf_\alpha F_{A_\alpha}(x \cdot y) \leq \inf_\alpha F_{A_\alpha}(y) = F_{\bigcup_\alpha A_\alpha}(y).$$

Let  $y \leq x$ . Then  $T_{A_\alpha}(y) \geq T_{A_\alpha}(x)$ ,  $I_{A_\alpha}(y) \geq I_{A_\alpha}(x)$ , and  $F_{A_\alpha}(y) \leq F_{A_\alpha}(x)$  for all  $\alpha$ . One can easily see that  $T_{\bigcup_\alpha A_\alpha}(y) \geq T_{\bigcup_\alpha A_\alpha}(x)$ ,  $I_{\bigcup_\alpha A_\alpha}(y) \geq I_{\bigcup_\alpha A_\alpha}(x)$ , and  $F_{\bigcup_\alpha A_\alpha}(y) \leq F_{\bigcup_\alpha A_\alpha}(x)$ . Therefore,  $\bigcup_\alpha A_\alpha$  is a single valued neutrosophic ideal of  $G$ .  $\square$

**Theorem 4.18.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVNS over  $X$ . Then  $A$  is a single valued neutrosophic subgroupoid of  $G$  if and only if  $L_{(\alpha, \beta, \gamma)}$  is either the empty set or a subgroupoid of  $G$  for all  $0 \leq \alpha, \beta, \gamma \leq 1$ .

*Proof.* Let  $A$  be a single valued neutrosophic subgroupoid of  $G$  and  $x, y \in L_{(\alpha, \beta, \gamma)} \neq \emptyset$ . Then  $T_A(x), T_A(y) \geq \alpha$ ,  $I_A(x), I_A(y) \geq \beta$ , and  $F_A(x), F_A(y) \leq \gamma$ . Since  $A$  is a single valued neutrosophic subgroupoid of  $G$ , it follows that  $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y) \geq \alpha$ ,  $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y) \geq \beta$ , and  $F_A(x \cdot y) \leq F_A(x) \vee F_A(y) \leq \gamma$ . Thus,  $x \cdot y \in L_{(\alpha, \beta, \gamma)}$ . Let  $y \leq x$  and  $x \in L_{(\alpha, \beta, \gamma)}$ . Then  $T_A(y) \geq T_A(x) \geq \alpha$ ,  $I_A(y) \geq I_A(x) \geq \beta$ , and  $F_A(y) \leq F_A(x) \leq \gamma$ . Thus,  $y \in L_{(\alpha, \beta, \gamma)}$  and hence,  $L_{(\alpha, \beta, \gamma)}$  is a subgroupoid of  $G$ .

Conversely, let  $L_{(\alpha, \beta, \gamma)} \neq \emptyset$  be a subgroupoid of  $G$  for all  $0 \leq \alpha, \beta, \gamma \leq 1$  and  $x, y \in G$  with  $N_A(x) = (\alpha_1, \beta_1, \gamma_1)$  and  $N_A(y) = (\alpha_2, \beta_2, \gamma_2)$ . By setting  $(\alpha, \beta, \gamma) = (\alpha_1 \wedge \alpha_2, \beta_1 \wedge \beta_2, \gamma_1 \vee \gamma_2)$ , we get that  $x, y \in L_{(\alpha, \beta, \gamma)}$ . Having  $L_{(\alpha, \beta, \gamma)} \neq \emptyset$  a subgroupoid of  $G$  implies that  $x \cdot y \in L_{(\alpha, \beta, \gamma)}$ . The latter implies that  $T_A(x \cdot y) \geq \alpha = T_A(x) \wedge T_A(y)$ ,  $I_A(x \cdot y) \geq \beta = I_A(x) \wedge I_A(y)$ , and  $F_A(x \cdot y) \leq \gamma = F_A(x) \vee F_A(y)$ . Let  $y \leq x$  with  $N_A(x) = (\alpha, \beta, \gamma)$ . Then  $y \in L_{(\alpha, \beta, \gamma)}$  and hence,  $T_A(y) \geq \alpha = T_A(x)$ ,  $I_A(y) \geq \beta = I_A(x)$ , and  $F_A(y) \leq \gamma = F_A(x)$ . Thus,  $A$  is a single valued neutrosophic subgroupoid of  $G$ .  $\square$

**Theorem 4.19.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $X$ . Then  $A$  is a single valued neutrosophic left (right) ideal of  $G$  if and only if  $L_{(\alpha, \beta, \gamma)}$  is either the empty set or a left (right) ideal of  $G$  for all  $0 \leq \alpha, \beta, \gamma \leq 1$ .

*Proof.* The proof is similar to that of Theorem 4.18.  $\square$

**Theorem 4.20.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $X$ . Then  $A$  is a single valued neutrosophic ideal of  $G$  if and only if  $L_{(\alpha, \beta, \gamma)}$  is either the empty set or an ideal of  $G$  for all  $0 \leq \alpha, \beta, \gamma \leq 1$ .

*Proof.* The proof follows from Theorem 4.19 and having an ideal of an ordered groupoid is a left ideal and right ideal of it.  $\square$

**Corollary 4.21.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $X$ . If  $A$  is a single valued neutrosophic left (right) ideal of  $G$  then  $A$  is a single valued neutrosophic subgroupoid of  $G$ .

*Proof.* The proof follows from Theorem 4.18 and Theorem 4.19.  $\square$

**Remark 4.22.** The converse of Corollary 4.21 may not hold. (See Example 4.11.)

## 4.2 Single valued neutrosophic filters of groupoids

**Definition 4.23.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  an SVN over  $G$ . Then  $A$  is single valued neutrosophic filter of  $G$  if for all  $x, y \in G$ , the following conditions hold:

1.  $T_A(x \cdot y) = T_A(x) \wedge T_A(y)$ ;
2.  $I_A(x \cdot y) = I_A(x) \wedge I_A(y)$ ;
3.  $F_A(x \cdot y) = F_A(x) \vee F_A(y)$ ;
4. If  $x \leq y$  then  $T_A(x) \leq T_A(y)$ ,  $I_A(x) \leq I_A(y)$ , and  $F_A(x) \geq F_A(y)$ .

**Example 4.24.** Let  $(G_4, \cdot_4, \leq_4)$  be the ordered groupoid defined in Example 3.12. Then

$$A = \left\{ \frac{1}{(0.1, 0.6, 1)}, \frac{2}{(0.1, 0.6, 1)}, \frac{3}{(0.9, 0.8, 0)} \right\}$$

is a single valued neutrosophic filter of  $G_4$ .

**Remark 4.25.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $\alpha, \beta, \gamma \in [0, 1]$  be fixed values. Then

$$A = \left\{ \frac{x}{(\alpha, \beta, \gamma)} : x \in G \right\}$$

is single valued neutrosophic filter of  $G$ . Moreover, it is called the **trivial single valued neutrosophic filter**.

**Lemma 4.26.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic filter of  $G$ . Then  $\bigcap_\alpha A_\alpha$  is a single valued neutrosophic filter of  $G$ .

*Proof.* The proof can be done in a similar way to that of Lemma 4.12.  $\square$

**Remark 4.27.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A_\alpha$  a single valued neutrosophic filter of  $G$ . Then  $\bigcup_\alpha A_\alpha$  may not be a single valued neutrosophic filter of  $G$ .

We illustrate Remark 4.13 by the following example.

**Example 4.28.** Let  $(G, \cdot)$  be the groupoid defined by Table 6.

Table 6: The groupoid  $(G, \cdot)$

$\cdot$	1	2	3
1	1	2	2
2	2	2	2
3	2	2	3

By setting  $\leq = \{(1, 1), (2, 2), (3, 3)\}$ , we get that  $(G, \cdot, \leq)$  is an ordered groupoid. By defining the SVNS  $A, B$  on  $G$  as follows.

$$N_A(1) = N_A(2) = (0.6, 0.8, 0.1), N_A(3) = (1, 0.8, 0.1);$$

$$N_B(1) = (1, 0.6, 0.4), N_B(2) = N_B(3) = (0.9, 0.6, 0.4),$$

we get that  $A, B$  are single valued neutrosophic filters of  $G$ . Since  $T_{A \cup B}(1 \cdot 3) = T_{A \cup B}(2) = 0.9 \neq 1 = T_{A \cup B}(1) \wedge T_{A \cup B}(3)$ , it follows that  $A \cup B$  is not a single valued neutrosophic filter of  $G$ .

**Lemma 4.29.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  a single valued neutrosophic set over  $G$ . If  $A$  is a single valued neutrosophic filter of  $G$  then for all  $0 \leq \alpha, \beta, \gamma \leq 1$ ,  $L_{(\alpha, \beta, \gamma)}$  is either the empty set or a filter of  $G$ .

*Proof.* Let  $A$  be a single valued neutrosophic filter of  $G$  and  $x, y \in G$ . One can easily see that if  $x, y \in L_{(\alpha, \beta, \gamma)} \neq \emptyset$  then  $x \cdot y \in L_{(\alpha, \beta, \gamma)}$ . If  $x \cdot y \in L_{(\alpha, \beta, \gamma)} \neq \emptyset$  then  $T_A(x \cdot y) \geq \alpha$ ,  $I_A(x \cdot y) \geq \beta$ , and  $F_A(x \cdot y) \leq \gamma$ . Since  $A$  is a single valued neutrosophic filter of  $G$ , it follows that  $T_A(x \cdot y) = T_A(x) \wedge T_A(y) \geq \alpha$ ,  $I_A(x \cdot y) = I_A(x) \wedge I_A(y) \geq \beta$ , and  $F_A(x \cdot y) = F_A(x) \vee F_A(y) \leq \gamma$ . The latter implies that  $T_A(x), T_A(y) \geq \alpha$ ,  $I_A(x), I_A(y) \geq \beta$ , and  $F_A(x), F_A(y) \leq \gamma$  and hence,  $x, y \in L_{(\alpha, \beta, \gamma)}$ . Let  $y \leq x$  and  $y \in L_{(\alpha, \beta, \gamma)}$ . Then  $\alpha \leq T_A(y) \leq T_A(x)$ ,  $\beta \leq I_A(y) \leq I_A(x)$ , and  $\gamma \geq F_A(y) \geq F_A(x)$ . Thus,  $x \in L_{(\alpha, \beta, \gamma)}$  and hence,  $L_{(\alpha, \beta, \gamma)}$  is a filter of  $G$ . Therefore,  $L_{(\alpha, \beta, \gamma)} \neq \emptyset$  is filter of  $G$ .  $\square$

**Lemma 4.30.** Let  $(G, \cdot, \leq)$  be an ordered groupoid and  $A$  a single valued neutrosophic set over  $G$ . If  $\bar{L}_{(\alpha, \beta, \gamma)} = \{x \in G : N_A(x) = (\alpha, \beta, \gamma)\}$  is a filter of  $G$  for all  $0 \leq \alpha, \beta, \gamma \leq 1$  then  $A$  is a single valued neutrosophic filter of  $G$ .

*Proof.* Let  $x, y \in G$  with  $N_A(x \cdot y) = (\alpha, \beta, \gamma)$ . Then  $x \cdot y \in \bar{L}_{(\alpha, \beta, \gamma)}$ . Having  $\bar{L}_{(\alpha, \beta, \gamma)}$  a filter of  $G$  implies that  $x, y \in \bar{L}_{(\alpha, \beta, \gamma)}$ . The latter implies that  $N_A(x) = N_A(y) = (\alpha, \beta, \gamma)$  and hence,  $T_A(x) \wedge T_A(y) = \alpha = T_A(x \cdot y)$ ,  $I_A(x) \wedge I_A(y) = \beta = I_A(x \cdot y)$ , and  $F_A(x) \vee F_A(y) = \gamma = F_A(x \cdot y)$ . Let  $x \leq y$  with  $N_A(x) = (\alpha, \beta, \gamma)$ . Having  $x \in \bar{L}_{(\alpha, \beta, \gamma)}$  and  $\bar{L}_{(\alpha, \beta, \gamma)}$  a filter of  $G$  implies that  $y \in \bar{L}_{(\alpha, \beta, \gamma)}$ . The latter implies that  $T_A(x) = \alpha \leq T_A(y)$ ,  $I_A(x) = \beta \leq I_A(y)$ , and  $F_A(x) = \gamma \geq F_A(y)$ . Therefore,  $A$  is a single valued neutrosophic filter of  $G$ .  $\square$

## 5 Some remarks on SVNS in ordered groups

In this section, we apply the definition of SVNS in ordered groupoids to ordered groups and point out some remarks and results about SVNS in ordered groups. Ideas of this section can be considered as a base for a new possible research on SVNS in ordered groups.

**Definition 5.1.** Let  $(G, \cdot, \leq)$  be an ordered group and  $A$  an SVNS over  $G$ . Then  $A$  is single valued neutrosophic subgroup of  $G$  if for all  $x, y \in G$ , the following conditions hold:

- (1)  $T_A(x \cdot y) \geq T_A(x) \wedge T_A(y)$ ;
- (2)  $I_A(x \cdot y) \geq I_A(x) \wedge I_A(y)$ ;
- (3)  $F_A(x \cdot y) \leq F_A(x) \vee F_A(y)$ ;
- (4)  $T_A(x^{-1}) \geq T_A(x)$ ,  $I_A(x^{-1}) \geq I_A(x)$ ,  $F_A(x^{-1}) \leq F_A(x)$ ;
- (5) If  $x \leq y$  then  $T_A(x) \geq T_A(y)$ ,  $I_A(x) \geq I_A(y)$ , and  $F_A(x) \leq F_A(y)$ .

**Proposition 5.2.** Let  $(G, \cdot, \leq)$  be an ordered group with identity “ $e$ ” and  $A$  a single valued neutrosophic subgroup of  $G$ . Then the following statements hold.

1.  $N_A(x) = N_A(x^{-1})$  for all  $x \in G$ .
2.  $T_A(e) \geq T_A(x)$ ,  $I_A(e) \geq I_A(x)$ , and  $F_A(e) \leq F_A(x)$  for all  $x \in G$ .

*Proof.* The proof is straightforward.  $\square$

**Proposition 5.3.** Let  $(G, \cdot, \leq)$  be an ordered group and  $A$  an SVNS over  $G$ . Then  $A$  is a single valued neutrosophic left (right) ideal of  $G$  if and only if  $A$  is the trivial single valued neutrosophic ideal of  $G$ .

*Proof.* The proof follows from Proposition 4.8.  $\square$

**Proposition 5.4.** Let  $(G, \cdot, \leq)$  be an ordered group with identity “ $e$ ” and  $A$  an SVN over  $G$ . If  $e$  and  $x$  are comparable for all  $x \in G$  then  $A$  is a single valued neutrosophic subgroup of  $G$  if and only if  $A$  is the trivial single valued neutrosophic subgroup of  $G$ .

*Proof.* If  $A$  is the trivial single valued neutrosophic subgroup of  $G$  then we are done.

Let  $A$  be a single valued neutrosophic subgroup of  $G$ . Since  $e, x$  are comparable, it follows that  $x \leq e$  or  $e \leq x$ . If  $x \leq e$  then  $T_A(x) \geq T_A(e)$ ,  $I_A(x) \geq I_A(e)$ , and  $F_A(x) \leq F_A(e)$ . Proposition 5.2. 2. implies that  $A$  is the trivial single valued neutrosophic subgroup of  $G$ . If  $e \leq x$  then  $x^{-1} \leq e$ . The latter implies that  $T_A(e) \geq T_A(x)$ ,  $I_A(e) \geq I_A(x)$ , and  $F_A(e) \leq F_A(x)$  and  $T_A(x) = T_A(x^{-1}) \geq T_A(e)$ ,  $I_A(x) = I_A(x^{-1}) \geq I_A(e)$ , and  $F_A(x) = F_A(x^{-1}) \leq F_A(e)$ . Thus,  $A$  is the trivial single valued neutrosophic subgroup of  $G$ .  $\square$

**Corollary 5.5.** Let  $(G, \cdot, \leq)$  be a total ordered group. Then  $G$  has no non-trivial single valued neutrosophic subgroups.

*Proof.* Since  $(G, \cdot, \leq)$  is a total ordered group, it follows that “ $e$ ” (the identity of  $G$ ) and  $x$  are comparable for all  $x \in G$ . Proposition 5.4 completes the proof.  $\square$

**Proposition 5.6.** Let  $(G, \cdot, \leq)$  be an ordered cyclic group with identity “ $e$ ” and generator  $a$ , and  $A$  an SVN over  $G$ . If  $e \leq a$  then  $A$  is a single valued neutrosophic subgroup of  $G$  if and only if  $A$  is the trivial single valued neutrosophic subgroup of  $G$ .

*Proof.* If  $A$  is the trivial single valued neutrosophic subgroup of  $G$  then we are done.

Let  $A$  be a single valued neutrosophic subgroup of  $G$ . Since  $e \leq a$ , it follows that  $a^{-1} \leq e$  and hence  $T_A(e) \leq T_A(a^{-1}) = T_A(a)$ ,  $I_A(e) \leq I_A(a^{-1}) = I_A(a)$ , and  $F_A(e) \geq F_A(a^{-1}) = F_A(a)$ . The latter and Proposition 5.2. 2. imply that  $T_A(e) = T_A(a)$ ,  $I_A(e) = I_A(a)$ , and  $F_A(e) = F_A(a)$ . Having  $e \leq a$  and  $(G, \cdot, \leq)$  an ordered group implies that  $e \leq a^k$  for all  $k = 1, 2, \dots$  and hence,  $a^{-k} \leq e$ . The latter implies that  $T_A(e) = T_A(a^k)$ ,  $I_A(e) = I_A(a^k)$ , and  $F_A(e) = F_A(a^k)$  for all  $k \in \mathbb{Z}$ . Therefore,  $A$  is the trivial single valued neutrosophic subgroup of  $G$ .  $\square$

**Example 5.7.** Using Proposition 5.6 we get that the ordered group of integers under standard addition and usual order has no non-trivial single valued neutrosophic subgroups.

**Proposition 5.8.** Let  $(G, \cdot, \leq)$  be a finite ordered group with identity “ $e$ ” and  $A$  an SVN over  $G$ . If  $e \leq a$  or  $a \leq e$  then  $a = e$ .

*Proof.* Let  $|G| = n$ . Then  $a^n = e$ . If  $e \leq a$  then  $a \leq a^k$  for all  $k = 1, 2, \dots$ . By setting  $k = n$ , we get that  $a \leq e$ . And if  $a \leq e$  then  $e \leq a^{-1}$  and hence  $a^{-1} \leq (a^{-1})^n = e$ . In both cases, we get that  $a = e$ .  $\square$

**Proposition 5.9.** Let  $(G, \cdot, \leq)$  be a finite ordered group with identity “ $e$ ”. Then “ $\leq$ ” is the trivial order on  $G$ .

*Proof.* Suppose that there exist  $x, y \in G$  such that  $x \leq y$ . Then  $e \leq yx^{-1}$ . Proposition 5.8 asserts that  $yx^{-1} = e$  and hence,  $x = y$ .  $\square$

From Proposition 5.9, we deduce that studying single valued neutrosophic subgroups of finite ordered group is the same as studying single valued neutrosophic subgroups of groups as the order is the trivial order. As a result, studying single valued neutrosophic subgroups of ordered groups should start with infinite groups.

## 6 Conclusion and discussion

This paper contributed to the study of neutrosophic algebraic structures by introducing, for the first time, SVN in ordered algebraic structures. Several new concepts were defined and studied like single valued neutrosophic subgroupoids, single valued neutrosophic ideals, and single valued neutrosophic filters of ordered groupoids and many interesting examples were presented. Finally, an application of this study to ordered groups was discussed. The latter can be considered as a base for a new possible research on SVN over ordered groups.

For future work, we will work on SVN in ordered groups and elaborate more properties about it. Also we will work on SVN in ordered semigroups.

**Funding:** “This research received no external funding.”

**Conflicts of Interest:** “The authors declare no conflict of interest.”

**Acknowledgement:** “The authors thank the reviewers for their valuable comments that improved the quality of the manuscript.”

## References

- [1] Al- Tahan, M., “Some Results on Single Valued Neutrosophic (Weak) Polygroups”, International Journal of Neutrosophic Science (IJNS), Vol 2, No. 1, pp. 38-46, 2020.
- [2] Al- Tahan, M. and Davvaz, B., “Neutrosophic  $\mathcal{N}$ -Ideals ( $\mathcal{N}$ -Subalgebras) of Subtraction Algebra”, International Journal of Neutrosophic Science (IJNS), Vol 3, No. 1, pp. 44-53, 2020.
- [3] Al-Tahan, M. and Davvaz, B., “On Single Valued Neutrosophic Sets and Neutrosophic  $\mathcal{N}$ -Structures: Applications on Algebraic Atructures (Hyperstructures)”, International Journal of Neutrosophic Science (IJNS), Vol 3, No. 2, pp. 108-117, 2020.
- [4] Atanassov, K.T, “Intuitionistic Fuzzy Sets”, Fuzzy Sets and Systems, Vol 20, No. 1, pp. 87-96, 1986.
- [5] Clifford, A.H and Preston, G.B, “The Algebraic Theory of Semigroups”, Vol 1, Amer. Math. Soc., Math. Surveys 7, Providence, Rhode Island, 1977.
- [6] Fuchs, L., “Partially Ordered Algebraic Systems”, Int. Ser. of Monographs on Pure and Appl. Math. 28, Pergamon Press, Oxford, 1963.
- [7] Kehayopulu, N. and Tsingelis, M., “Fuzzy sets in Ordered Groupoids”, Semigroup Forum, Vol 65, pp. 128-132, 2002.
- [8] Khan, M., Anis, S., Smarandache, and F., Jun, Y. B., “Neutrosophic N-structures and their Applications in Semigroups”, Annals of Fuzzy Mathematics and Informatics, Vol 14, No. 6, pp. 583-598, 2017.
- [9] Rezaei, A. and Smarandache. F., “On Neutro-BE-algebras and Anti-BE-algebras”, International Journal of Neutrosophic Science (IJNS), Vol 4, No. 1, pp. 8-15, 2020.
- [10] Smarandache. F., “Neutrosophy: Neutrosophic Probability, Set and logic”, Ann Arbor, Michigan, USA, Vol 105, 2002.
- [11] Smarandache. F., “Neutrosophic Set- A Generalization of the Intuitionistic Fuzzy Set”, Int. J. Pure Appl. Math., Vol 24, pp. 287-297, 2005.
- [12] Smarandache. F., “NeutroAlgebra is a Generalization of Partial Algebra”, International Journal of Neutrosophic Science (IJNS), Vol 2, No. 1, pp. 8-17, 2020.
- [13] Ulucay and V., Sahin, M., “Neutrosophic Multigroups and Applications”, Mathematics, Vol 7, 95, 2019.
- [14] Wang, H., Smarandache, F., Zhang, Y. and Sunderraman, R., “Single Valued Neutrosophic Sets”, Multi-space and Multi-structure, Vol 4, pp. 410-413, 2010.
- [15] Zadeh, L., “Fuzzy Sets”, Inform and Control, Vol 8, pp. 338-353, 1965.



## On Finite NeutroGroups of Type-NG[1,2,4]

A.A.A. Agboola<sup>◇</sup>

Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria.

agboolaaaa@funaab.edu.ng

◇ In commemoration of the 60th birthday of the author

### Abstract

The NeutroGroups as alternatives to the classical groups are of different types with different algebraic properties. In this paper, we are going to study a class of NeutroGroups of type-NG[1,2,4]. In this class of NeutroGroups, the closure law, the axiom of associativity and existence of inverse are taking to be either partially true or partially false for some elements; while the existence of identity element and axiom of commutativity are taking to be totally true for all the elements. Several examples of NeutroGroups of type-NG[1,2,4] are presented along with their basic properties. It is shown that Lagrange's theorem holds for some NeutroSubgroups of a NeutroGroup and failed to hold for some NeutroSubgroups of the same NeutroGroup. It is also shown that the union of two NeutroSubgroups of a NeutroGroup can be a NeutroSubgroup even if one is not contained in the other; and that the intersection of two NeutroSubgroups may not be a NeutroSubgroup. The concepts of NeutroQuotientGroups and NeutroGroupHomomorphisms are presented and studied. It is shown that the fundamental homomorphism theorem of the classical groups is holding in the class of NeutroGroups of type-NG[1,2,4].

**Keywords:** NeutroGroup; AntiGroup; NeutroSubgroup; NeutroQuotientGroup; NeutroGroupHomomorphism.

## 1 Introduction and Preliminaries

In any classical algebraic structure  $(X, *)$ , the law of composition of the elements of  $X$  otherwise called a binary operation  $*$  is well defined for all the elements of  $X$  that is,  $x * y \in X \quad \forall x, y \in X$ ; and axioms like associativity, commutativity, distributivity, etc. defined on  $X$  with respect to  $*$  are totally true for all the elements of  $X$ . The compositions of elements of  $X$  this way are restrictive and do not reflect the reality. It does not give room for compositions that are either partially defined, partially undefined (indeterminate), and partially outer-defined or totally outerdefined with respect to  $*$ . However in the domain of knowledge, science and reality, the law of composition and axioms defined on  $X$  may either be only partially defined (partially true), or partially undefined (partially false), or totally undefined (totally false) with respect to  $*$ . In an attempt to model the reality by allowing the law of composition on  $X$  to be either partially defined, partially undefined (indeterminate), and partially outerdefined or totally outerdefined, Smarandache [8] in 2019 introduced the notions of NetroDefined and AntiDefined laws, as well as the notions of NeutroAxiom and AntiAxiom inspired by his work in [9], which has given birth to new fields of research called NeutroStructures and AntiStructures. For any classical algebraic law or axiom defined on  $X$ , there correspond neutrosophic triplets  $\langle \text{Law}, \text{NeutroLaw}, \text{AntiLaw} \rangle$  and  $\langle \text{Axiom}, \text{NeutroAxiom}, \text{AntiAxiom} \rangle$  respectively. Smarandache in [7] studied NeutroAlgebras and AntiAlgebras and in [6], he studied Partial Algebras, Universal Algebras, Effect Algebras and Boole's Partial Algebras and he showed that NeutroAlgebras are generalization of Partial Algebras. Rezaei and Smarandache [5] studied Neutro-BE-algebras and Anti-BE-algebras and fundamentally they showed that any classical algebra  $S$  with  $n$  operations (laws and axioms) where  $n \geq 1$  will have  $(2^n - 1)$  NeutroAlgebras and  $(3^n - 2^n)$  AntiAlgebras. Agboola et al. in [1] studied NeutroAlgebras and AntiAlgebras viz-a-viz the classical number systems  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$  and in [2], he studied NeutroGroups by considering three NeutroAxioms (NeutroAssociativity, existence of NeutroNeutral element and existence of NeutroInverse element). In addition, he studied NeutroSubgroups, NeutroCyclicGroups, NeutroQuotientGroups and NeutroGroupHomomorphisms. He showed that generally, Lagrange's theorem and 1st isomorphism theorem of the classical groups do not

hold in the class of NeutroGroups. In [3], Agboola studied NeutroRings by considering three NeutroAxioms (NeutroAbelianGroup (additive), NeutroSemigroup (multiplicative) and NeutroDistributivity (multiplication over addition)). He presented Several results and examples on NeutroRings, NeutroSubgrings, NeutroIdeals, NeutroQuotientRings and NeutroRingHomomorphisms. He showed that the 1st isomorphism theorem of the classical rings holds in the class of NeutroRings. Motivated and inspired by the work of Rezaei and Smarandache in [5], the work on NeutroGroups presented in [2] is revisited and the present work is devoted to the study of a class of NeutroGroups of type-NG[1,2,4]. In this class of NeutroGroups, the closure law, the axiom of associativity and existence of inverse are taking to be either partially true or partially false for some elements; while the existence of identity element and axiom of commutativity are taking to be totally true for all the elements. Several examples of NeutroGroups of type-NG[1,2,4] are presented along with their basic properties. It is shown that Lagrange's theorem holds for some NeutroSubgroups of a NeutroGroup and failed to hold for some NeutroSubgroups of the same NeutroGroup. It is also shown that the union of two NeutroSubgroups of a NeutroGroup can be a NeutroSubgroup even if one is not contained in the other; and that the intersection of two NeutroSubgroups may not be a NeutroSubgroup. The concepts of NeutroQuotientGroups and NeutroGroupHomomorphisms are presented and studied. It is shown that the fundamental homomorphism theorem of the classical groups is holding in the class of NeutroGroups of type-NG[1,2,4].

**Definition 1.1.** [6]

- (i) A classical operation is an operation well defined for all the set's elements.
- (ii) A NeutroOperation is an operation partially well defined, partially indeterminate, and partially outer defined on the given set.
- (iii) An AntiOperation is an operation that is outer defined for all set's elements.
- (iv) A classical law/axiom defined on a nonempty set is a law/axiom that is totally true (i.e. true for all set's elements).
- (v) A NeutroLaw/NeutroAxiom (or Neutrosophic Law/Neutrosophic Axiom) defined on a nonempty set is a law/axiom that is true for some set's elements [degree of truth (T)], indeterminate for other set's elements [degree of indeterminacy (I)], or false for the other set's elements [degree of falsehood (F)], where  $T, I, F \in [0, 1]$ , with  $(T, I, F) \neq (1, 0, 0)$  that represents the classical axiom, and  $(T, I, F) \neq (0, 0, 1)$  that represents the AntiAxiom.
- (vi) An AntiLaw/AntiAxiom defined on a nonempty set is a law/axiom that is false for all set's elements.
- (vii) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements, and false for other elements), and no AntiOperation or AntiAxiom.
- (viii) An AntiAlgebra is an algebra endowed with at least one AntiOperation or at least one AntiAxiom.

**Theorem 1.2.** [5] Let  $\mathbb{U}$  be a nonempty finite or infinite universe of discourse and let  $S$  be a finite or infinite subset of  $\mathbb{U}$ . If  $n$  classical operations (laws and axioms) are defined on  $S$  where  $n \geq 1$ , then there will be  $(2^n - 1)$  NeutroAlgebras and  $(3^n - 2^n)$  AntiAlgebras.

## 2 Main Results

**Definition 2.1.** [Classical group][4]

Let  $G$  be a nonempty set and let  $*$  :  $G \times G \rightarrow G$  be a binary operation on  $G$ . The couple  $(G, *)$  is called a classical group if the following conditions hold:

- (G1)  $x * y \in G \forall x, y \in G$  [closure law].
- (G2)  $x * (y * z) = (x * y) * z \forall x, y, z \in G$  [axiom of associativity].
- (G3) There exists  $e \in G$  such that  $x * e = e * x = x \forall x \in G$  [axiom of existence of neutral element].
- (G4) There exists  $y \in G$  such that  $x * y = y * x = e \forall x \in G$  [axiom of existence of inverse element] where  $e$  is the neutral element of  $G$ .

If in addition  $\forall x, y \in G$ , we have

(G5)  $x * y = y * x$ , then  $(G, *)$  is called an abelian group.

**Definition 2.2.** [Neutrosophication of the law and axioms of the classical group]

- (NG1) There exist at least three duplets  $(x, y), (u, v), (p, q) \in G$  such that  $x * y \in G$  (degree of truth T) and  $[u * v = \text{outer-defined/indeterminate (degree of indeterminacy I) or } p * q \notin G]$  (degree of falsehood F) [NeutroClosureLaw].
- (NG2) There exist at least three triplets  $(x, y, z), (p, q, r), (u, v, w) \in G$  such that  $x * (y * z) = (x * y) * z$  (degree of truth T) and  $[p * (q * r)] \text{ or } [(p * q) * r] = \text{outer-defined/indeterminate (degree of indeterminacy I) or } u * (v * w) \neq (u * v) * w]$  (degree of, falsehood F) [NeutroAxiom of associativity (NeutroAssociativity)].
- (NG3) There exists an element  $e \in G$  such that  $x * e = e * x = x$  (degree of truth T) and  $[[x * e] \text{ or } [e * x] = \text{outer-defined/indeterminate (degree of indeterminacy I) or } x * e \neq x \neq e * x]$  (degree of falsehood F) for at least one  $x \in G$  [NeutroAxiom of existence of neutral element (NeutroNeutralElement)].
- (NG4) There exists an element  $u \in G$  such that  $x * u = u * x = e$  (degree of truth T) and  $[[x * u] \text{ or } [u * x] = \text{outer-defined/indeterminate (degree of indeterminacy I) or } x * u \neq e \neq u * x]$  for at least one  $x \in G$  (degree of falsehood F) [NeutroAxiom of existence of inverse element (NeutroInverseElement)] where  $e$  is a NeutroNeutralElement in  $G$ .
- (NG5) There exist at least three duplets  $(x, y), (u, v), (p, q) \in G$  such that  $x * y = y * x$  (degree of truth T) and  $[[u * v] \text{ or } [v * u] = \text{outer-defined/indeterminate (degree of indeterminacy I) or } p * q \neq q * p]$  (degree of falsehood F) [NeutroAxiom of commutativity (NeutroCommutativity)].

**Definition 2.3.** [AntiSophication of the law and axioms of the classical group]

- (AG1) For all the duplets  $(x, y) \in G, x * y \notin G$  [AntiClosureLaw].
- (AG2) For all the triplets  $(x, y, z) \in G, x * (y * z) \neq (x * y) * z$  [AntiAxiom of associativity (AntiAssociativity)].
- (AG3) There does not exist an element  $e \in G$  such that  $x * e = e * x = x \forall x \in G$  [AntiAxiom of existence of neutral element (AntiNeutralElement)].
- (AG4) There does not exist  $u \in G$  such that  $x * u = u * x = e \forall x \in G$  [AntiAxiom of existence of inverse element (AntiInverseElement)] where  $e$  is an AntiNeutralElement in  $G$ .
- (AG5) For all the duplets  $(x, y) \in G, x * y \neq y * x$  [AntiAxiom of commutativity (AntiCommutativity)].

**Definition 2.4.** [NeutroGroup]

A NeutroGroup  $NG$  is an alternative to the classical group  $G$  that has at least one NeutroLaw or at least one of  $\{NG1, NG2, NG3, NG4\}$  with no AntiLaw or AntiAxiom.

**Definition 2.5.** [AntiGroup]

An AntiGroup  $AG$  is an alternative to the classical group  $G$  that has at least one AntiLaw or at least one of  $\{AG1, AG2, AG3, AG4\}$ .

**Definition 2.6.** [NeutroAbelianGroup]

A NeutroAbelianGroup  $NG$  is an alternative to the classical abelian group  $G$  that has at least one NeutroLaw or at least one of  $\{NG1, NG2, NG3, NG4\}$  and  $NG5$  with no AntiLaw or AntiAxiom.

**Definition 2.7.** [AntiAbelianGroup]

An AntiAbelianGroup  $AG$  is an alternative to the classical abelian group  $G$  that has at least one AntiLaw or at least one of  $\{AG1, AG2, AG3, AG4\}$  and  $AG5$ .

**Proposition 2.8.** Let  $(G, *)$  be a finite or infinite classical non abelian group. Then:

- (i) there are 15 types of NeutroNonAbelianGroups,
- (ii) there are 65 types of AntiNonAbelianGroups.

*Proof.* It follows from Theorem [1.2](#). □

**Proposition 2.9.** Let  $(G, *)$  be a finite or infinite classical abelian group. Then:

- (i) there are 31 types of NeutroAbelianGroups,

(ii) there are 211 types of AntiAbelianGroups.

*Proof.* It follows from Theorem 1.2. □

**Remark 2.10.** It is evident from Theorem 2.8 and Theorem 2.9 that there are many types of NeutroGroups and NeutroAbelianGroups. The type of NeutroGroups studied by Agboola in [2] is that for which  $G1, G2, G3, G4$  and  $G5$  are either partially true or partially false.

**Definition 2.11.** Let  $(NG, *)$  be a NeutroGroup.  $NG$  is said to be finite of order  $n$  if the cardinality of  $NG$  is  $n$  that is  $o(NG) = n$ . Otherwise,  $NG$  is called an infinite NeutroGroup and we write  $o(NG) = \infty$ .

**Definition 2.12.** Let  $(AG, *)$  be an AntiGroup.  $AG$  is said to be finite of order  $n$  if the cardinality of  $AG$  is  $n$  that is  $o(AG) = n$ . Otherwise,  $AG$  is called an infinite AntiGroup and we write  $o(AG) = \infty$ .

**Example 2.13.** Let  $NG = \mathbb{N} = \{1, 2, 3, 4, \dots\}$ . Then  $(NG, \cdot)$  is a finite NeutroGroup where " $\cdot$ " is the binary operation of ordinary multiplication.

**Example 2.14.** Let  $AG = \mathbb{Q}_+^*$  be the set of all irrational positive numbers and consider algebraic structure  $(AG, *)$  where  $*$  is ordinary multiplication of numbers. It is clear that  $*$  is a total AntiLaw defined on  $AG$ . The binary operation  $*$  is totally associative for all the triplets  $(x, y, z)$  with  $x, y, z \in AG$ . There is no neutral element(s) for all the elements  $AG$  and hence no element of  $AG$  has an inverse. Finally, the operation  $*$  is commutative for all the duplets  $(x, y)$  with  $x, y \in AG$ . Hence by Definition 2.5,  $(AG, *)$  is an infinite AntiGroup.

**Example 2.15.** Let  $\mathbb{U} = \{a, b, c, d, e, f\}$  be a universe of discourse and let  $AG = \{a, b, c, \}$  be a subset of  $\mathbb{U}$ . Let  $*$  be a binary operation defined on  $AG$  as shown in the Cayley table below:

*	a	b	c
a	d	c	b
b	c	e	a
c	b	a	f

It is clear from the table that except for the compositions  $a * a = d, b * b = e, c * c = f$  that are outer-defined with the degree of falsity 33%, the rest compositions are inner-defined with 66.7% degree of truth. This shows that  $G1$  is partially true and partially false so that  $*$  is a NeutroLaw. Also,  $G2$  is partial true and partially false. There are  $3^3 = 27$  possible triplets out of which only 6 can verify associativity of  $*$ . Hence degree of associativity of  $*$  is 22.2% while the degree of non-associativity is 77.8% so that  $*$  is NeutroAssociative. However,  $G3$  and  $G4$  are totally false for all the elements of  $AG$  which shows that  $AG3$  and  $AG4$  are satisfied. Lastly,  $G5$  is partially true with the degree of truth 50% and partially false with 50% degree of falsity which shows that  $*$  is NeutroCommutative. Hence by Definition 2.5,  $(AG, *)$  is a finite AntiGroup.

### 3 Certain Types of NeutroGroups

In this section, we are going to study certain types of NeutroGroups  $(NG, *)$ . The NeutroGroups will be named according to which of  $NG1 - NG5$  is(are) satisfied. In the sequel,  $x * y$  will be written as  $xy \forall x, y \in NG$ .

**Example 3.1.** Let  $\mathbb{U} = \{a, b, c, d, e, f\}$  be a universe of discourse and let  $NG = \{e, a, b, c\}$  be a subset of  $\mathbb{U}$ . Let  $*$  be a binary operation defined on  $NG$  as shown in the Cayley table below:

*	e	a	b	c
e	e	a	b	c
a	a	b	a	b
b	b	c	f	c
c	c	d	c	e

It is clear from the table that  $G1, G2, G3, G4$  and  $G5$  are partially true and partially false with respect to  $*$  as shown below:

- (i) **NeutroClosureLaw (NG1):** Except for the compositions  $b * b = f, c * a = d$  which are false with 12.5% degree of falsity, all other compositions are true with 87.5% degree of truth.

(ii) **NeuroAssociativity (NG2):**

$$\begin{aligned} a * (b * c) &= (a * b) * c = b. \\ a * (a * a) &= a, \text{ but } (a * a) * a = c \neq a. \end{aligned}$$

(iii) **NeuroNeutralElement (NG3):**

$$\begin{aligned} N_e &= N_a = N_b = e \text{ but} \\ N_c &= e \text{ or } b. \end{aligned}$$

(iv) **NeuroInverseElement (NG4):**

$$\begin{aligned} I_e &= e, \\ I_a &\text{ does not exist,} \\ I_b &\text{ does not exist,} \\ I_c &= e. \end{aligned}$$

(v) **NeuroCommutativity (NG5):**

$$\begin{aligned} b * c &= c * b = c. \\ a * b &= a \text{ but } b * a = c \neq a. \end{aligned}$$

We have just shown that  $(NG, *)$  is a finite NeuroAbelianGroup. This is an example of the class of NeuroGroups studied by Agboola in [2]. This class of NeuroGroups are referred to as of type-NG[1,2,3,4,5].

**Example 3.2.** Let  $\mathbb{U} = \{a, b, c, d, e, f\}$  be a universe of discourse and let  $NG = \{a, b, c, e\}$  be a subset of  $\mathbb{U}$ . Let  $*$  be a binary operation defined on  $NG$  as shown in the Cayley table below:

*	a	b	c	e
a	e	c	f	a
b	c	e	d	b
c	d	a	e	c
e	a	b	c	e

It is clear from the table that  $G3$  and  $G4$  are totally true for all the elements of  $NG$ . However,  $G1$ ,  $G2$  and  $G5$  are partially true and partially false with respect to  $*$  as shown below:

- (i) **NeuroClosureLaw (NG1):** Except for the compositions  $a * c = f, b * c = d, c * a = d$  which are outer-defined with 18.75% degree of falsity, all other compositions are inner-defined with 81.25% degree of truth.

(ii) **NeuroAssociativity (NG2):**

$$\begin{aligned} c * (b * b) &= (c * b) * b = c. \\ a * (b * c) &= a * d = \text{outer-defined}, (a * b) * c = e. \end{aligned}$$

(iii) **NeuroCommutativity (NG5):**

$$\begin{aligned} a * b &= b * a = c. \\ a * c &= f \text{ but } c * a = d \neq f. \end{aligned}$$

We have just shown that  $(NG, *)$  is a finite NeuroAbelianGroup of type-NG[1,2,5].

**Example 3.3.** Let  $NG = \mathbb{Z}$  and let  $*$  be a binary operation defined on  $NG$  by

$$x * y = x + xy \quad x, y \in \mathbb{Z}.$$

It is clear that only  $G1$  is totally true for all  $x, y \in NG$  but  $G2, G3, G4$  and  $G5$  are partially true and partially false with respect to  $*$  as shown below:

(i) **NeuroAssociativity (NG2):**

$$\begin{aligned}
 x * (y * z) &= x + xy + xyz \\
 (x * y) * z &= x + xy + xz + xyz \quad \text{by equating, we have} \\
 x + xy + xyz &= x + xy + xz + xyz \\
 \Rightarrow xz &= 0 \quad \text{from which we obtain} \\
 x &= 0 \quad \text{or} \quad z = 0.
 \end{aligned}$$

This shows that only the triplets  $(0, y, 0)$ ,  $(0, y, z)$ ,  $(x, y, 0)$  can verify associativity of  $*$ .

(ii) **NeuroNeutralElement (NG3):**

It is clear that only the element  $0 \in NG$  has 0 as its neutral element and no neutral(s) for other elements of  $NG$ .

(iii) **NeuroInverseElement (NG4):**

Again, only the element  $0 \in NG$  has 0 as the inverse element and no inverse(s) for other elements of  $NG$ .

(iv) **NeuroCommutativity (NG5):**

Only the duplet  $(0, 0)$  can verify the commutativity of  $*$  and not any other duplet(s)  $(x, y)$ . Hence,  $(NG, *)$  is an infinite NeuroAbelianGroup of type-NG[2,3,4,5].

**Definition 3.4.** Let  $(NG, *)$  be a NeuroGroup. A nonempty subset  $NH$  of  $NG$  is called a NeuroSubgroup of  $NG$  if  $(NH, *)$  is also a NeuroGroup of the same type as  $NG$ . If  $(NH, *)$  is a NeuroGroup of a type different from that of  $NG$ , then  $NH$  will be called a QuasiNeuroSubgroup of  $NG$ .

**Example 3.5.** Let  $(NG, *)$  be the NeuroGroup of Example 3.2 and let  $NH_1 = \{a, c, e\}$  and  $NH_2 = \{b, c, e\}$  be two subsets of  $NG$ . Let  $*$  be defined on  $NH_1$  and  $NH_2$  as shown in the Cayley tables below:

$NH_1 :$	*	$a$	$c$	$e$	$NH_2 :$	*	$b$	$c$	$e$
	$a$	$e$	$f$	$a$		$b$	$e$	$d$	$b$
	$c$	$d$	$e$	$c$		$c$	$a$	$e$	$c$
	$e$	$a$	$c$	$e$		$e$	$b$	$c$	$e$

It can easily be shown that  $(NH_1, *)$  and  $(NH_2, *)$  are NeuroGroups of type-NG[1,2,5] and therefore  $NH_1$  and  $NH_2$  are NeuroSubgroups of  $NG$ . We note that  $o(NG) = 4$ ,  $o(NH_1) = 3 = o(NH_2)$ . Since 3 does not divide 4, it follows that Lagrange's theorem does not hold. Now consider the following:

$$\begin{aligned}
 NH_1 \cup NH_2 &= \{a, b, c, e\} = NG. \\
 NH_1 \cap NH_2 &= \{c, e\}.
 \end{aligned}$$

These show that  $NH_1 \cup NH_2$  is a NeuroSubgroup of  $NG$  but  $NH_1 \cap NH_2$  is not a Neurosubgroup of  $NG$ . It is however observed that  $NH_1 \cap NH_2$  is a group as can be seen in the Cayley table below:

$NH_1 \cap NH_2 :$	<table><tr><td>*</td><td><math>c</math></td><td><math>e</math></td></tr><tr><td><math>c</math></td><td><math>e</math></td><td><math>c</math></td></tr><tr><td><math>e</math></td><td><math>c</math></td><td><math>e</math></td></tr></table>	*	$c$	$e$	$c$	$e$	$c$	$e$	$c$	$e$
	*	$c$	$e$							
	$c$	$e$	$c$							
$e$	$c$	$e$								

**Definition 3.6.** Let  $(NG, *)$  be a NeuroGroup and let  $a \in NG$  be a fixed element.

(i) The center of  $NG$  denoted by  $Z(NG)$  is a set defined by

$$Z(NG) = \{x \in NG : xg = gx \text{ for at least one } g \in NG\}.$$

(ii) The centralizer of  $a \in G$  denoted by  $NC_a$  is a set defined by

$$NC_a = \{g \in NG : ga = ag\}.$$

**Example 3.7.** Let  $(NG, *)$  be the NeuroGroup of Example 3.2. Then:

(i)  $Z(NG) = \{a, b, c, e\} = NG$ . This shows that  $Z(NG)$  is a NeuroSubgroup of  $NG$ .(ii)  $NC_a = \{a, b, e\}$ ,  $NC_b = \{a, b, e\}$ ,  $NC_c = \{c, e\}$  and  $NC_e = \{a, b, c, e\}$ . We have that  $NC_a$  and  $NC_b$  are not NeuroSubgroups of  $NG$ ,  $NC_c$  is a group and  $NC_e$  is a NeuroSubgroup of  $NG$ .

## 4 Characterization of Finite NeutroGroups of type-NG[1,2,4]

In this section, we are going to study finite NeutroGroups of type-NG[1,2,4] that is NeutroGroups  $(NG, *)$  where  $G3$  and  $G5$  are totally true for all the elements of  $NG$  and where  $G1$ ,  $G2$  and  $G4$  are either partially true or partially false for some elements of  $NG$ .

**Example 4.1.** Let  $NG = \{1, 2, 3, 4\} \subseteq \mathbb{Z}_5$  and let  $*$  be a binary operation on  $NG$  defined as

$$x * y = x + y + 4 \quad \forall x, y \in NG.$$

Then  $(NG, *)$  is a finite NeutroGroup of type-NG[1,2,4] as can be seen in the Cayley table

*	1	2	3	4
1	1	2	3	4
2	2	3	4	0
3	3	4	0	1
4	4	0	1	2

**Example 4.2.** Let  $NG = \{1, 2, 3\} \subseteq \mathbb{Z}_4$  and let  $\bullet$  be a binary operation defined on  $NG$  as shown in the Cayley table

$\star$	1	2	3
1	1	2	3
2	2	0	2
3	3	2	1

Then  $(NG, \bullet)$  is a finite NeutroGroup of type-NG[1,2,4].

**Example 4.3.** Let  $NK = \{1, 3, 5\} \subseteq \mathbb{Z}_8$  and let  $\circ$  be a binary operation define on  $NK$  as shown in the Cayley table

$\circ$	1	3	5
1	1	3	5
3	3	1	7
5	5	7	1

Then  $(NK, \circ)$  is a finite NeutroGroup of type-NG[1,2,4].

**Example 4.4.** Let  $NG = \{1, 2, 3, 4, 5\} \subseteq \mathbb{Z}_{10}$  and let  $*$  be a binary operation define on  $NG$  as shown in the Cayley table

*	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	0
3	3	6	9	2	5
4	4	8	2	6	0
5	5	0	5	0	5

Then  $(NG, *)$  is a finite NeutroGroup of type-NG[1,2,4].

**Example 4.5.** Let  $(NG, \bullet)$  and  $(NK, \circ)$  be NeutroGroups of Examples [4.2](#) and [4.3](#) respectively. Then

$$\begin{aligned} NG \times NG &= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}, \\ NK \times NK &= \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}, \\ NG \times NK &= \{(1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}. \end{aligned}$$

It can easily be shown that  $(NG \times NG, \bullet)$ ,  $(NK \times NK, \circ)$  and  $(NG \times NK, \square)$  are NeutroGroups of type-NG[1,2,4].

**Proposition 4.6.** Let  $(NG, \bullet)$  and  $(NK, \circ)$  be any two NeutroGroups of the same type-NG[1,2,4]. Then  $(NG \times NG, \bullet)$ ,  $(NK \times NK, \circ)$  and  $(NG \times NK, \square)$  are NeutroGroups of type-NG[1,2,4].

*Proof.* The proof is easy and so omitted. □

**Proposition 4.7.** Let  $(NG, *)$  be a finite NeutroGroup of type-NG[1,2,4]. The classical laws of indices do not hold.

*Proof.* Since  $*$  is a NeutroLaw and  $*$  is NeutroAssociative over  $NG$ , the result follows.  $\square$

**Corollary 4.8.** No NeutroGroup  $(NG, *)$  of type-NG[1,2,4] can be cyclic that is, generated by an element  $x \in NG$ .

**Proposition 4.9.** Let  $(NG, *)$  be a finite NeutroGroup of type-NG[1,2,4]. If  $x$  and  $y$  are invertible elements of  $NG$ , then

- (i)  $(x^{-1})^{-1} = x$ .
- (ii)  $(xy)^{-1} = x^{-1}y^{-1}$ .

*Proof.* Obvious.  $\square$

**Example 4.10.** Let  $(NG, *)$  be the NeutroGroup of Example 4.1 and consider the following subsets of  $NG$ .

$$NH_1 = \{1, 2\}, NH_2 = \{1, 3\}, NH_3 = \{1, 4\}, NH_4 = \{1, 2, 3\}, NH_5 = \{1, 2, 4\}, NH_6 = \{1, 3, 4\}.$$

It can be shown that  $(NH_i, *)$ ,  $i = 1, 2, 3, 4, 5, 6$  are NeutroSubgroups of  $NG$ . Next consider the following:

$$\begin{aligned} NH_1 \cup NH_2 &= NH_1 \cup NH_4 = \{1, 2, 3\} \quad [\text{NeutroSubgroups of NG}]. \\ NH_1 \cup NH_3 &= NH_1 \cup NH_5 = NH_3 \cup NH_5 = \{1, 2, 4\} \quad [\text{NeutroSubgroups of NG}]. \\ NH_2 \cup NH_3 &= NH_2 \cup NH_6 = \{1, 3, 4\} \quad [\text{NeutroSubgroups of NG}]. \\ NH_3 \cup NH_4 &= NH_5 \cup NH_6 = \{1, 2, 3, 4\} \quad [\text{trivial NeutroSubgroups of NG}]. \\ NH_1 \cap NH_4 &= NH_1 \cap NH_5 = NH_4 \cap NH_5 = \{1, 2\} \quad [\text{NeutroSubgroups of NG}]. \\ NH_2 \cap NH_4 &= NH_2 \cap NH_6 = NH_4 \cap NH_6 = \{1, 3\} \quad [\text{NeutroSubgroups of NG}]. \\ NH_3 \cap NH_5 &= NH_3 \cap NH_6 = \{1, 4\} \quad [\text{NeutroSubgroups of NG}]. \end{aligned}$$

$$NH_1 \cap NH_2 = NH_2 \cap NH_3 = \{1\} \quad [\text{not NeutroSubgroups of NG}].$$

**Example 4.11.** Let  $(NG, *)$  be the NeutroGroup of Example 4.4 and consider the following subsets of  $NG$ .

$$NH_1 = \{1, 2, 4, 5\}, NH_2 = \{1, 2, 3, 5\}.$$

It can be shown that  $(NH_i, *)$ ,  $i = 1, 2$  are NeutroSubgroups of  $NG$ . Next consider the following:

$$\begin{aligned} NH_1 \cup NH_2 &= \{1, 2, 3, 4, 5\} \quad [\text{a NeutroSubgroup of NG}]. \\ NH_1 \cap NH_2 &= \{1, 2, 3, 4, 5\} \quad [\text{a trivial NeutroSubgroup of NG}]. \end{aligned}$$

**Remark 4.12.** Examples 4.10 and 4.11 have shown that in the NeutroGroups of type-NG[1,2,4], we can have the following:

- (i) Lagrange's theorem may hold for some NeutroSubgroups of the NeutroGroups and fail to hold for some NeutroSubgroups.
- (ii) The union of two NeutroSubgroups of the NeutroGroups can be NeutroSubgroups even if one is not contained in the other.
- (iii) The intersection of two NeutroSubgroups of the NeutroGroups can be NeutroSubgroups.

**Definition 4.13.** Let  $NH$  be a NeutroSubgroup of the NeutroGroup  $(NG, *)$  and let  $x \in NG$ .

- (i)  $xNH$  the left coset of  $NH$  in  $NG$  is defined by

$$xNH = \{xh : h \in NH\}.$$

- (ii) The number of distinct left cosets of  $NH$  in  $NG$  is called the index of  $NH$  in  $NG$  denoted by  $[NG : NH]$ .

(iii) The set of all distinct left cosets of  $NH$  in  $NG$  denoted by  $NG/NH$  is defined by

$$NG/NH = \{xNH : x \in NG\}.$$

**Example 4.14.** Let  $(NG, *)$  be the NeutroGroup of Example 4.1 and let  $(NH_i, *)$ ,  $i = 1, 2, 3, 4, 5, 6$  be the NeutroSubgroups of Example 4.10. The left cosets of  $NH_i$  are computed as follows.

$$\begin{aligned} 1NH_1 &= \{1, 2\}, 2NH_1 = \{2, 3\}, 3NH_1 = \{3, 4\}, 4NH_1 = \{0, 4\}. \\ 1NH_2 &= \{1, 3\}, 2NH_2 = \{2, 4\}, 3NH_2 = \{0, 3\}, 4NH_2 = \{1, 4\}. \\ 1NH_3 &= \{1, 4\}, 2NH_3 = \{0, 2\}, 3NH_3 = \{1, 3\}, 4NH_3 = \{2, 4\}. \\ 1NH_4 &= \{1, 2, 3\}, 2NH_4 = \{2, 3, 4\}, 3NH_4 = \{0, 3, 4\}, 4NH_4 = \{0, 1, 4\}. \\ 1NH_5 &= \{1, 2, 4\}, 2NH_5 = \{0, 2, 3\}, 3NH_5 = \{1, 3, 4\}, 4NH_5 = \{0, 2, 4\}. \\ 1NH_6 &= \{1, 3, 4\}, 2NH_6 = \{0, 2, 4\}, 3NH_6 = \{0, 1, 3\}, 4NH_6 = \{1, 2, 4\}. \\ \therefore NG/NH_i &= \{1NH_i, 2NH_i, 3NH_i, 4NH_i\}, i = 1, 2, 3, 4, 5, 6. \end{aligned}$$

**Example 4.15.** Let  $(NG, *)$  be the NeutroGroup of Example 4.4 and let  $(NH_i, *)$ ,  $i = 1, 2$  be the NeutroSubgroups of Example 4.11. The left cosets of  $NH_i$  are computed as follows.

$$\begin{aligned} 1NH_1 &= \{1, 2, 4, 5\}, 2NH_1 = \{0, 2, 4, 8\}, 3NH_1 = \{2, 3, 5, 6\}, 4NH_1 = \{0, 4, 6, 8\}, 5NH_1 = \{0, 5\}. \\ 1NH_2 &= \{1, 2, 3, 5\}, 2NH_2 = \{0, 2, 4, 6\}, 3NH_2 = \{3, 5, 6, 9\}, 4NH_2 = \{0, 2, 4, 8\}, 5NH_2 = \{0, 5\}. \\ \therefore NG/NH_i &= \{1NH_i, 2NH_i, 3NH_i, 4NH_i, 5NH_i\}, i = 1, 2. \end{aligned}$$

**Lemma 4.16.** Let  $NH$  be a NeutroSubgroup of the NeutroGroup  $(NG, *)$  of type-NG[1,2,4] and let  $x \in NG$ . Then,  $xNH = NH$  if and only if  $x = e$  where  $e$  is the identity element in  $NG$ .

*Proof.* Obvious. □

**Remark 4.17.** Examples 4.14 and 4.15 have shown that in the NeutroGroups of type-NG[1,2,4] distinct left cosets of NeutroSubgroups in the NeutroGroups do not necessarily partition the NeutroGroups.

Let  $NH$  be a NeutroSubgroup of a NeutroGroup  $(NG, *)$  of type-NG[1,2,4] and let  $NG/NH$  be the set of distinct left cosets of  $NH$  in  $NG$ . For  $xNH, yNH \in NG/NH$  with  $x, y \in NG$ , let  $\odot$  be a binary operation defined on  $NG/NH$  by

$$xNH \odot yNH = xyNH \quad \forall x, y \in NG.$$

We want to investigate if the couple  $(NG/NH, \odot)$  is a NeutroGroup of type-NG[1,2,4] using the following examples.

**Example 4.18.** Let  $NG/NH_i = \{1NH_i, 2NH_i, 3NH_i, 4NH_i\}$ ,  $i = 1, 2, 3, 4, 5, 6$  be as given in Example 4.14. For  $i = 1$ , we have

$\odot$	$1NH_1$	$2NH_1$	$3NH_1$	$4NH_1$
$1NH_1$	$1NH_1$	$2NH_1$	$3NH_1$	$4NH_1$
$2NH_1$	$2NH_1$	$3NH_1$	$4NH_1$	$0NH_1$
$3NH_1$	$3NH_1$	$4NH_1$	$0NH_1$	$1NH_1$
$4NH_1$	$4NH_1$	$0NH_1$	$1NH_1$	$2NH_1$

It is clear from the Cayley table that  $(NG/NH_1, \odot)$  is a NeutroGroup of type-NG[1,2,4] with  $1NH_1$  as the identity element. This is also true for  $i = 2, 3, 4, 5, 6$ .

**Example 4.19.** Let  $NG/NH_i = \{1NH_i, 2NH_i, 3NH_i, 4NH_i, 5NH_i\}$ ,  $i = 1, 2$  be as given in Example 4.15. For  $i = 1$ , we have

$\odot$	$1NH_1$	$2NH_1$	$3NH_1$	$4NH_1$	$5NH_1$
$1NH_1$	$1NH_1$	$2NH_1$	$3NH_1$	$4NH_1$	$5NH_1$
$2NH_1$	$2NH_1$	$4NH_1$	$6NH_1$	$8NH_1$	$0NH_1$
$3NH_1$	$3NH_1$	$6NH_1$	$9NH_1$	$2NH_1$	$5NH_1$
$4NH_1$	$4NH_1$	$8NH_1$	$2NH_1$	$6NH_1$	$0NH_1$
$5NH_1$	$5NH_1$	$0NH_1$	$5NH_1$	$0NH_1$	$5NH_1$

It is evident from the Cayley table that  $(NG/NH_1, \odot)$  is a NeutroGroup of type-NG[1,2,4] with  $1NH_1$  as the identity element.. This is also true for  $i = 2$ .

**Remark 4.20.** It is evident from Examples 4.18 and 4.19 that if  $NH$  is a NeutroSubgroup of the NeutroGroup of type-NG[1,2,4], then  $NG/NH$  the set of distinct left cosets can be made a NeutroGroup of type-NG[1,2,4] by defining appropriate binary operation  $\odot$  on  $NG/NH$ . The NeutroGroup  $NG/NH$  is called the Quotient-NeutroGroup of  $NG$  factored by  $NH$ .

**Definition 4.21.** Let  $(NG, *)$  and  $(NH, \circ)$  be any two NeutroGroups of type-NG[1,2,4]. The mapping  $\phi : NG \rightarrow NH$  is called a NeutroGroupHomomorphism if  $\phi$  preserves the binary operations  $*$  and  $\circ$  that is if for at least a duplet  $(x, y) \in G$ , we have

$$\phi(x * y) = \phi(x) \circ \phi(y).$$

The Kernel of  $\phi$  denoted by  $Ker\phi$  is defined by

$$Ker\phi = \{x : \phi(x) = e_{NH}\}$$

where  $e_{NH}$  is the identity element in  $NH$ .

The Image of  $\phi$  denoted by  $Im\phi$  is defined by

$$Im\phi = \{y \in H : y = \phi(x) \text{ for some } x \in NG\}.$$

If in addition  $\phi$  is a NeutroBijection, then  $\phi$  is called a NeutroGroupIsomorphism and we write  $NG \cong NH$ . NeutroGroupEpimorphism, NeutroGroupMonomorphism, NeutroGroupEndomorphism, and NeutroGroupAutomorphism are similarly defined.

**Example 4.22.** Let  $(NG, \bullet)$  be the NeutroGroup of Example 4.2 and let  $\phi : NG \times NG \rightarrow NG$  be a projection given by

$$\phi(x, y) = y \quad \forall x, y \in NG.$$

Then

$$\phi(1, 1) = \phi(2, 1) = \phi(3, 1) = 1, \phi(1, 2) = \phi(2, 2) = \phi(3, 2) = 2, \phi(1, 3) = \phi(2, 3) = \phi(3, 3) = 3.$$

Since  $\phi((2, 1)(3, 3)) = \phi(2, 3) = 3$  and  $\phi(2, 1)\phi(3, 3) = 1 \bullet 3 = 3$  but  $\phi((2, 2)(2, 3)) = \phi(0, 2) = ?$  and  $\phi(2, 2)\phi(2, 3) = 2 \bullet 3 = 2$ , it follows that  $\phi$  is a NeutroGroupHomomorphism.  $Im\phi = \{1, 2, 3\} = NG$  and  $Ker\phi = \{(1, 1), (2, 1), (3, 1)\}$ . The  $Ker\phi$  is a NeutroSubgroup of  $NG \times NG$  as can be seen in the following Cayley table

$\bullet$	(1, 1)	(2, 1)	(3, 1)
(1, 1)	(1, 1)	(2, 1)	(3, 1)
(2, 1)	(2, 1)	(0, 1)	(2, 1)
(3, 1)	(3, 1)	(2, 1)	(1, 1)

It is evident from the table that  $(Ker\phi, \bullet)$  is a NeutroGroup of type-NG[1,2,4] and since  $Ker\phi \subseteq NG \times NG$ , it follows that  $ker\phi$  is a NeuroSubgroup.

**Example 4.23.** Let  $(NK, \circ)$  be the NeutroGroup of Example 4.3 and let  $\psi : NK \times NK \rightarrow NK$  be a projection given by

$$\psi(x, y) = x \quad \forall x, y \in NK.$$

Then

$$\psi(1, 1) = \psi(1, 3) = \psi(1, 5) = 1, \psi(3, 1) = \psi(3, 3) = \psi(3, 5) = 3, \psi(5, 1) = \psi(5, 3) = \psi(5, 5) = 5.$$

Since  $\psi((1, 1)(1, 3)) = \psi(1, 3) = 1$  and  $\psi(1, 1)\psi(1, 3) = 1 \circ 1 = 1$  but  $\psi((1, 5)(5, 3)) = \psi(5, 7) = ?$  and  $\psi(1, 5)\psi(5, 3) = 1 \circ 5 = 5$ , it follows that  $\psi$  is a NeutroGroupHomomorphism.  $Im\psi = \{1, 3, 5\} = NK$  and  $Ker\psi = \{(1, 1), (1, 3), (1, 5)\}$ . The  $Ker\psi$  is a NeutroSubgroup of  $NK \times NK$  as can be seen in the following Cayley table

$\circ$	(1, 1)	(1, 3)	(1, 5)
(1, 1)	(1, 1)	(1, 3)	(1, 5)
(1, 3)	(1, 3)	(1, 1)	(1, 7)
(1, 5)	(1, 5)	(1, 7)	(1, 1)

It is evident from the table that  $(Ker\psi, \circ)$  is a NeutroGroup of type-NG[1,2,4] and since  $Ker\psi \subseteq NK \times NK$ , it follows that  $ker\psi$  is a NeuroSubgroup.

**Example 4.24.** Let  $NG/NH_i = \{1NH_i, 2NH_i, 3NH_i, 4NH_i\}, i = 1, 2, 3, 4, 5, 6$  be the NeutroQuotient-Group of Example 4.18. For  $i = 1$ , let  $\psi : NG \rightarrow NG/NH_1$  be a mapping defined by

$$\psi(x) = xNH_1 \quad \forall x \in NG.$$

From Example 4.14 we have

$$\psi(1) = 1NH_1 = \{1, 2\}, \psi(2) = 2NH_1 = \{2, 3\}, \psi(3) = 3NH_1 = \{3, 4\}, \psi(4) = 4NH_1 = \{0, 4\}.$$

Next,

$$\begin{aligned} \psi(2 * 3) &= \psi(4) = 4NH_1 = \{0, 4\}, \\ \psi(2) \odot \psi(3) &= 2NH_1 \odot 3NH_1 = 2 * 3NH_1 = 4NH_1 = \{0, 4\} \quad \text{but then,} \\ \psi(2 * 4) &= \psi(0) = ? \\ \psi(2) \odot \psi(4) &= 2NH_1 \odot 4NH_1 = 2 * 4NH_1 = 0NH_1 = ?. \end{aligned}$$

This shows that  $\psi$  is a NeutroGroupHomomorphism. The  $Ker\psi = 1NH_1 = \{1, 2\}$  the identity element of  $NG/NH_1$ .

**Example 4.25.** Let  $NG/NH_i = \{1NH_i, 2NH_i, 3NH_i, 4NH_i, 5NH_i\}, i = 1, 2$  be the NeutroQuotient-Group of Example 4.19. For  $i = 1$ , let  $\phi : NG \rightarrow NG/NH_1$  be a mapping defined by

$$\phi(x) = xNH_1 \quad \forall x \in NG.$$

From Example 4.15 we have

$$\begin{aligned} \psi(1) &= 1NH_1 = \{1, 2, 4, 5\}, \phi(2) = 2NH_1 = \{0, 2, 4, 8\}, \phi(3) = 3NH_1 = \{2, 3, 5, 6\}, \\ \phi(4) &= 4NH_1 = \{0, 4, 6, 8\}, \phi(5) = 5NH_1 = \{0, 5\}. \end{aligned}$$

Next,

$$\begin{aligned} \phi(3 * 5) &= \phi(5) = 5NH_1 = \{0, 5\}, \\ \phi(3) \odot \phi(5) &= 3NH_1 \odot 5NH_1 = 3 * 5NH_1 = 5NH_1 = \{0, 5\} \quad \text{but then,} \\ \phi(3 * 2) &= \phi(6) = ? \\ \phi(3) \odot \phi(2) &= 3NH_1 \odot 2NH_1 = 3 * 2NH_1 = 6NH_1 = ?. \end{aligned}$$

This shows that  $\psi$  is a NeutroGroupHomomorphism. The  $Ker\psi = 1NH_1 = \{1, 2, 4, 5\}$  the identity element of  $NG/NH_1$ .

**Proposition 4.26.** Let  $(NG, *)$  and  $(NH, \odot)$  be NeutroGroups of type-NG[1,2,4] and let  $e_{NG}$  and  $e_{NH}$  be identity elements in  $NG$  and  $NH$  respectively. Suppose that  $\phi : NG \rightarrow NH$  is a NeutroGroupHomomorphism. Then:

- (i)  $\phi(e_{NG}) = e_{NH}$ .
- (ii)  $\phi(x^{-1}) = (\phi(x))^{-1}$  for every invertible element  $x \in NG$ .
- (iii)  $Ker\phi$  is a NeutroSubgroup of  $NG$ .
- (iv)  $Im\phi$  is a NeutroSubgroup of  $NH$ .
- (v)  $\phi$  is NeutroInjective if and only if  $Ker\phi = \{e_{NG}\}$ .

*Proof.* The same as for the classical groups and so omitted. □

**Proposition 4.27.** Let  $NH$  be a NeutroSubgroup of a NeutroGroup  $(NG, *)$  of type-NG[1,2,4]. The mapping  $\psi : NG \rightarrow NG/NH$  defined by

$$\psi(x) = xNH \quad \forall x \in NG$$

is a NeutroGroupHomomorphism and the  $Ker\psi = NH$ .

*Proof.* The same as for the classical groups and so omitted. □

**Proposition 4.28.** Let  $\phi : NG \rightarrow NH$  be a NeutroGroupHomomorphism and let  $NK = Ker\phi$ . Then the mapping  $\psi : NG/NK \rightarrow Im\phi$  defined by

$$\psi(xNK) = \phi(x) \quad \forall x \in NG$$

is a NeutroGroupIsomorphism.

*Proof.* The same as for the classical groups and so omitted. □

## 5 Conclusion

We have in this work studied a class of NeutroGroups  $(NG, *)$  of type-NG[1,2,4]. In this class of NeutroGroups, the closure law, the axiom of associativity and existence of inverse were taking to be either partially true or partially false for some elements of  $NG$ ; while the existence of identity element and axiom of commutativity were taking to be totally true for all the elements of  $NG$ . Several examples of NeutroGroups of type-NG[1,2,4] were presented along with their basic properties. It was shown that Lagrange's theorem holds for some NeutroSubgroups of a NeutroGroup and failed to hold for some NeutroSubgroups of the same NeutroGroup. It was also shown that the union of two NeutroSubgroups of a NeutroGroup can be a NeutroSubgroup even if one is not contained in the other; and that the intersection of two NeutroSubgroups may not be a NeutroSubgroup. The concepts of NeutroQuotientGroups and NeutroGroupHomomorphisms were presented and studied. It was shown that the fundamental homomorphism theorem of the classical groups is holding in the class of NeutroGroups of type-NG[1,2,4]. We hope to study AntiGroups, revisit NeutroRings, study AntiRings, NeutroVectorSpaces, AntiVectorSpaces, NeutroModules, AntiModules, NeutroHypergroups, AntiHypergroups, NeutroHyperrings, AntiHyperrings, NeutroHypervectorSpaces and AntiHypervectorSpaces in our future papers.

## 6 Acknowledgment

The author is very grateful to Professor Florentin Smarandache for his private discussions, comments and suggestions during the preparation of this work.

## References

- [1] Agboola, A.A.A., Ibrahim, M.A. and Adeleke, E.O., "Elementary Examination of NeutroAlgebras and AntiAlgebras viz-a-viz the Classical Number Systems", International Journal of Neutrosophic Science, vol. 4 (1), pp. 16-19, 2020. DOI:10.5281/zenodo.3752896.
- [2] Agboola, A.A.A., "Introduction to NeutroGroups", International Journal of Neutrosophic Science (IJNS), Vol. 6 (1), pp. 41-47, 2020. (DOI: 10.5281/zenodo.3840761).
- [3] Agboola, A.A.A., "Introduction to NeutroRings", International Journal of Neutrosophic Science (IJNS), Vol. 7 (2), pp. 62-73, 2020. (DOI:10.5281/zenodo.3877121).
- [4] Gilbert, L. and Gilbert, J., "Elements of Modern Algebra", Eighth Edition, Cengage Learning, USA, 2015.
- [5] Rezaei, A. and Smarandache, F., "On Neutro-BE-algebras and Anti-BE-algebras (revisited)", International Journal of Neutrosophic Science (IJNS), Vol. 4 (1), pp. 08-15, 2020. (DOI: 10.5281/zenodo.3751862).
- [6] Smarandache, F., "NeutroAlgebra is a Generalization of Partial Algebra", International Journal of Neutrosophic Science, vol. 2 (1), pp. 08-17, 2020.
- [7] Smarandache, F., "Introduction to NeutroAlgebraic Structures and AntiAlgebraic Structures (revisited)", Neutrosophic Sets and Systems (NSS), vol. 31, pp. 1-16, 2020. DOI: 10.5281/zenodo.3638232.
- [8] Smarandache, F., "Introduction to NeutroAlgebraic Structures", in Advances of Standard and Nonstandard Neutrosophic Theories, Pons Publishing House Brussels, Belgium, Ch. 6, pp. 240-265, 2019.
- [9] Smarandache, F. Neutrosophy, Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998.  
<http://fs.unm.edu/eBook-Neutrosophic6.pdf> (edition online).



## A Review of Fuzzy Soft Topological Spaces, Intuitionistic Fuzzy Soft Topological Spaces and Neutrosophic Soft Topological Spaces

<sup>1</sup>M. Parimala, <sup>2</sup>M.Karthika, <sup>3</sup>F. Smarandache

<sup>1,2</sup>Department of Mathematics, Bannari Amman Institute of Technology, TN, India-638 401.

<sup>3</sup>University of New Mexico, Gallup, NM 87301, USA

Email<sup>1</sup>rishwanthpari@gmail.com, Email<sup>2</sup>karthikamuthusamy1991@gmail.com,  
Email<sup>3</sup>fsmarandache@gmail.com

### Abstract

The notion of fuzzy sets initiated to overcome the uncertainty of an object. Fuzzy topological space, intuitionistic fuzzy sets in topological structure space, vagueness in topological structure space, rough sets in topological space, theory of hesitancy and neutrosophic topological space, etc. are the extension of fuzzy sets. Soft set is a family of parameters which is also a set. Fuzzy soft topological space, intuitionistic fuzzy soft and neutrosophic soft topological space are obtained by incorporating soft sets with various topological structures. This motivates to write a review and study on various soft set concepts. This paper shows the detailed review of soft topological spaces in various sets like fuzzy, Intuitionistic fuzzy set and neutrosophy. Eventually, we compared some of the existing tools in the literature for easy understanding and exhibited their advantages and limitations.

**Keywords:** Soft sets, fuzzy soft topological space, intuitionistic fuzzy soft topological space, neutrosophic soft topological space.

### 1 Introduction

In the year 1999, Molodtsov<sup>[47]</sup> proposed the concept of soft sets (SS). This concept developed to overcome the difficulty to fix membership for each case. SS is a family of parameterization of the universe of discourse. Parameters may be numbers, meaningful words, sentences, etc. Anyone could define the parameterization for their convenient. This technique is very useful to model the uncertainties. Also, Molodtsov defined some basic operations and presented some uses of SS, such as stability and operations research, etc.

The first definition of soft spaces was introduced by the authors Shabir and Naz<sup>[70]</sup> and it is defined on the universe of discourse with a fixed set of parameters. Also they proved that a soft topological space provides a parameterized family of topological spaces. The researchers<sup>[14, 21, 52, 74]</sup> are developed the concept of soft set theory.

Fuzzy set (FS) was introduced by Zadeh<sup>[81]</sup> in the year 1965, every element 'a' in A has a membership value, where A is mapped from the universe of discourse to [0, 1]. Later Chang<sup>[24]</sup> introduced the concept fuzzy topology in the literature, which satisfies the three axioms of topology and also Chang used the same notation in fuzzy topology as Zadeh used for FS. After few years, Lower<sup>[39]</sup> defined fuzzy topology which is different from definition by Chang. Maji et al.<sup>[42]</sup> proposed the concept of fuzzy soft set (FSS) and defined some basic operations. Later, Tanay et al.<sup>[72]</sup> introduced fuzzy soft topological space (FSTS) and established the basic definitions of FSTS by incorporating the fuzzy topology and soft set. FSTS was applied in various ways say, game theory, analysis, etc. Fuzzy soft set in topological space further studied by Roy.<sup>[61, 63]</sup> The authors<sup>[10, 22, 30, 36, 45, 50, 54]</sup> are successfully applied FSTS in real life.

FS failed to address the rejection of an object in the set. So Atanassov<sup>[12]</sup> proposed the theory where every object in a set has both acceptance and rejection with subject to the constraint that sum of acceptance and rejection should not exceed 1 and non-negative and that theory is called Intuitionistic fuzzy set theory. Intuitionistic fuzzy set A(a) is an generalization of fuzzy set where every element a in A is a subset of universal set have degree of membership and degree of non- membership and each function map from the universe of

discourse to the interval  $[0, 1]$ . Researchers developed the theory by generalising it and got new result through extension.<sup>[6][15][16][57][59]</sup> Later, Maji et al.<sup>[43]</sup> introduced the notion of intuitionistic fuzzy soft set (IFSS). D. Coker<sup>[27]</sup> initiated the concept of IFSTS and followed by<sup>[34][53][76]</sup> developed the concept in decision making.

Atanassov failed to address the problem when indeterminacy occurs in the object. To address the difficulty, Smarandache<sup>[67][68]</sup> originated the concept called neutrosophic set. Every element in the neutrosophic set has truth, indeterminacy and falsity values respectively and which are maps from universe of discourse to  $[0, 1]$  with the constraint that truth, indeterminacy and falsity values should not exceed 3 and not less than zero under addition. Many complex problems in statistics, in graph theory when relationship between the object have acceptance, rejection and also indeterminacy, physics, image processing, networking and in decision making which can't be solved by existing classical methods. The generalisation of this notion also exist in the literature, namely neutrosophic soft topology, neutrosophic nano topology, neutrosophic nano ideals topology, neutrosophic support soft set,<sup>[56]</sup> neutrosophic soft supra topological spaces in various sets, etc. Maji et al.<sup>[41]</sup> presented the concepts of neutrosophic soft set. Maji<sup>[44]</sup> successfully applied the concept of neutrosophic soft sets (NSS) in pattern recognition, reasoning etc. Thereafter, Bera<sup>[18][19]</sup> initiated the concept of neutrosophic soft topological space (NSTS). The following authors<sup>[7][8][19][28][66]</sup> are developed NSTS. In this work, soft sets in various topological spaces are studied in detail. Advantages and limitations of different soft topological spaces are presented. Eventually comparison table of classical soft topological space, FSTS, IFSTS, NSTS are also presented.

## 2 Preliminaries

This section is a collection of definitions initiated by<sup>[12][18][27][40][43][64][67][72][81]</sup> for the development of soft sets in various uncertainty sets.

**Definition 2.1.** Fuzzy set  $A(a)$  of universal set  $X$  is defined by  $A = \{(a, \mu_A(a)) : a \in X\}$ , where  $\mu_A$  represent the degree of membership and it is mapped from the universal set  $X$  to the unit interval  $[0, 1]$ .

**Definition 2.2.** The pair  $(F, A)$  is called a FSS over  $X$ , where the mapping  $F : A \rightarrow F(U)$ ,  $F(U)$  is the set of all fuzzy subsets of non-empty set  $X$  and  $A$  is a subset of the set of parameters  $E$ .

**Definition 2.3.** Let the pair  $(X, \tau)$  be a FSTS and  $\tau$  be a family of FSS over  $X \neq \emptyset$ . The pair  $(X, \tau)$  is said to be a FSTS if it satisfying the following conditions: (i)  $0_E, 1_E \in \tau$ . (ii) If  $f_{A_1}, f_{A_2}$  in  $\tau$ , then  $f_{A_1} \wedge f_{A_2}$  in  $\tau$ . (ii) If  $(f_A)_i$  in  $\tau$ , for all  $i$  in  $J$ , then union of  $(f_A)_i$  in  $\tau$ . Then  $\tau$  is called a topology of fuzzy soft sets on  $X$ .

**Definition 2.4.** Let  $(X, \tau)$  be FSTS and  $f_A \in F(X, E)$ . The closure of fuzzy soft set  $f_A$  is intersection of all fuzzy soft closed supersets of  $f_A$ .

**Definition 2.5.** Let  $(X, \tau)$  be FSTS and  $f_A \in F(X, E)$ . The interior of fuzzy soft set  $f_A$  is union of all fuzzy soft open subsets of  $f_A$ .

**Definition 2.6.** Intuitionistic fuzzy set  $A(a)$  on the non-empty set  $X$  is defined by  $A = \{(a, \mu_A(a), \nu_A(a)) : a \in X\}$ , where  $\mu_A$  denotes truth value and  $\nu_A$  denote the falsehood and the map of truth value and falsehood from the universal set  $X$  to the interval  $[0, 1]$  and satisfying the constraint that sum of truth and falsehood value is lies between 0 and 1, for each  $a \in X$ .

**Definition 2.7.** Let  $X$  and  $E$  be the initial universe and set of all parameters respectively and  $A$  is a subset of the parameter set  $E$ . Let  $IF(U)$  be the set of all power set of  $X$ . If the mapping  $F$  from the set  $A$  to  $IF(U)$ , then the pair  $(F, A)$  is said to be intuitionistic soft set over  $X$ .

**Definition 2.8.** Neutrosophic set  $A(a)$  on the non-empty set  $X$  is defined by  $A = \{(a, \mu_A(a), \sigma_A(a), \nu_A(a)) : a \in X\}$ , where  $\mu_A$  represent the degree of membership,  $\sigma_A$  represent the degree of indeterminacy and  $\nu_A$  represent the degree of non-membership, indeterminacy and non-membership from the universal set  $X$  to the interval  $[0, 1]$  and with the constraint  $0 \leq \mu_A(a) + \sigma_A(a) + \nu_A(a) \leq 3$ , for each  $a \in X$ .

**Definition 2.9.** Let  $X$  be a non-empty set and the parameters set be  $E$ . The power set of  $X$  is denoted by  $P(X)$  and is defined as the collection of all neutrosophic set. The pair  $(F, I)$  is called the NSS over  $X$ , where  $I$  is a subset of  $X$  and the map  $F$  from  $I$  to  $P(X)$ .

**Definition 2.10.** Let  $X$  and  $E$  are the non-empty set and set of parameters respectively and NSTS  $(X, \tau)$  is a subset of NSS  $(X, E)$  satisfying the following:

(1) Null and universal soft set are the members of  $\tau$ . (2) Finite intersection of member of any finite sub collection of  $\tau$  also in  $\tau$ . (3) Arbitrary union of member of any sub collection of  $\tau$  also a member of  $\tau$ .

### 3 Soft Topological Spaces in Various Sets

#### 3.1 Fuzzy soft set

In 2008, Yao<sup>[73]</sup> presented the concept of soft fuzzy set and this concept tested for the significance of existing soft fuzzy set. Lastly, FSS relations and soft fuzzy set relations are compared with some example. Cagman<sup>[23]</sup> modified the definition of FSS and studied the concept with some of its properties. Finally, fuzzy soft aggregation operator is defined for effective construction of decision process.

Generalized FSS introduced by Majumdar<sup>[46]</sup> in 2010. Some properties of generalized FSS and its applications are presented by Manjundar. Tanay<sup>[72]</sup> first introduced the concept of FSTS to the literature. The authors also defined the notion of neighbourhood, family of neighborhood, interior and closure of FSS, Basis for FSTS and finally subspace of FSTS along with the some properties. Gunduz. C<sup>[31]</sup> defined interior, closure of FSTS. Further, Gunduz introduced open and closed sets in respect of FSS and continuous mapping in FSS, homeomorphism of FSTS. Characteristics of fuzzy soft topological structure also discussed. In 2011, Zhi Kong et al.<sup>[82]</sup> discussed FSS to present a real life problem with grey relation analysis theory. The result is verified with some cases. Mahanta<sup>[48]</sup> introduced and studied fuzzy soft point and its neighbourhood in a FSTS. Closure and interior of FSS are studied and investigated separation axioms and connectedness of FSTS. Abd El – Latif<sup>[4]</sup> developed and studied the concept of pre-connected, pre-separated, pre-soft subspace of FSTS. Generally, Pre - disconnectedness of FSTS is not traditional property proved by Abd El – Latif.<sup>[4]</sup> In 2012, Varol<sup>[73]</sup> brought the notion of fuzzy soft continuity and projection mapping of FSTS. Simsekler<sup>[67]</sup> defined fuzzy soft open sets and fuzzy soft closed sets in FSTS and also fuzzy Q-neighbourhood of fuzzy soft points are defined. Roy et al.<sup>[61,63]</sup> defined the concept of accumulation point using Q-neighbourhood and also proved that separation axioms exists using Q-neighbourhood in FSS.

Yang et al.<sup>[77]</sup> combined the concept of multi-fuzzy set and soft set to produce a new result called multi-fuzzy soft set. Also defined some theoretic operations say, union, intersection and complementary. Yang et al.<sup>[77]</sup> developed an algorithm using multi-fuzzy soft set. Eventually using the proposed algorithm, decision making problem is analysed. Roy and Maji<sup>[62]</sup> analysed the decision making problems using fuzzy soft sets. They construct an algorithm for selecting object from universe of discourse by considering maximum value among the score using score function. Cetkin<sup>[25]</sup> established the concept of continuous mappings in FSTS and presented the idea of anti-chain and isomorphism to FSTS.

In 2015, Kandil<sup>[37]</sup> introduced the concept of semi connected set, semi s-connected set, semi separated set in FSTS. Sabir Hussain<sup>[64]</sup> defined the soft pre-open set, soft alpha-open set in FSS and studied soft neighbourhood at fuzzy soft points. Also introduced soft regular open set and studied further. Finally, the relationships between the above proposed concepts are presented. Pre-open, pre-closed set of FSTS introduced by Abd El-Latif<sup>[13]</sup> and studied some properties of pre-regular, pre-normal space of FSTS. Fuzzy  $\alpha$ -connected set, fuzzy  $\alpha$ -separated set, fuzzy  $\alpha$ -S-connected set in FSTS established by A.M. Abd El-Latif.<sup>[2]</sup>

A. Kandil et al.<sup>[36]</sup> defined fuzzy soft point based on equivalence classes in the year 2015 and described that Universal fuzzy soft set can be written as the union of disjoint connected component. G. Kalpana et al.<sup>[35]</sup> introduced fuzzy soft r-open and fuzzy soft r-closed mappings, fuzzy soft r-closure, fuzzy soft r-interior, fuzzy soft r-continuous mapping through fuzzy soft set. Abd El-Latif<sup>[3]</sup> initiated the notion of  $\beta$ -open soft sets and  $\beta$ -separation axioms in FSTS and established the properties of  $\beta$ -closure and  $\beta$  – regular,  $\beta$ -normal space in FSTS.

#### 3.2 Intuitionistic Fuzzy Soft Set

Here, we present the initialization, extensions and generalization of intuitionistic fuzzy soft set in topological structure. Yang<sup>[76]</sup> originated the concept of interval - valued IFSS, defined the set theoretic operations and finally decision making problem solved by adopting existing algorithm. Mukherjee<sup>[49]</sup> proposed and studied a new type of sequence of intuitionistic fuzzy soft multi sets and some of its properties are investigated. Also the increasing, decreasing and convergent sequences of intuitionistic fuzzy soft multi- topological spaces are introduced by Mukherjee<sup>[49]</sup> Finally, cluster intuitionistic fuzzy soft multi topological space and their properties are studied. In 2010, Xu<sup>[74]</sup> presented the concept of IFSS by merging K.Atanassov intuitionistic fuzzy set and soft set. Developed some basic operations and applying this tool to target the type recognition problem. Jiang et al.<sup>[33]</sup> combined the two classical methods viz. soft set and interval-valued intuitionistic fuzzy set and produced a new result called interval-valued IFSS. Union, intersection and complement of interval-valued IFSS defined and established some basic properties.

In 2012 Yin et al.<sup>[79]</sup> introduced further the concept of IFSS. In particular, theoretical operations such as union, intersection and complement, etc. are introduced. Mapping on IFSS introduced and their basic properties also presented. Li et al.<sup>[40]</sup> proposed the novel notion called IFSTS in the year 2013. The author

also defined the interior, closure, base, relative complement and absolute IFSS and IFSTS. Some properties of IFSTS also presented.

In 2013, Agarwal et al.<sup>[5]</sup> developed the concept of generalized IFSS and this developed a generalized parameter to pool the intuitionistic fuzzy numbers. The author has developed three different algorithms mainly for decision making. One is for in medical diagnosis to compare the intuitionistic fuzzy numbers and remaining for measure the similarity, if any in selecting the supplier. Kumud Borgohain<sup>[38]</sup> studied IFSTS and defined intuitionistic fuzzy soft separation axioms, normal space and finally completely normal space of IFSTS. Osmanoglu et al.<sup>[55]</sup> introduced intuitionistic fuzzy soft finer and coarser topological space, Intuitionistic fuzzy soft discrete topology and intuitionistic fuzzy soft indiscrete topology. Further, soft points and complement of intuitionistic fuzzy soft points and separation axioms of the same introduced and their properties also studied. Cetkin<sup>[26]</sup> introduced the definition of closure intuitionistic supra fuzzy soft topological space.

In 2014, Bayramov. S<sup>[13]</sup> introduced the basic definitions of IFSTS namely, null and absolute IFSTS, interior and closure, associated closure of IFSTS. Some basic properties also investigated. Mukherjee<sup>[51]</sup> established the notion of intuitionistic fuzzy soft multi topological space for the parameterized family and also established the basic structure of intuitionistic fuzzy soft multi topological structure.

Shuker Mahmood<sup>[71]</sup> studied and established soft b-closed, soft b-continuous mapping, soft b- closed disconnectedness of IFSTS. In 2017, Yogalakshmi<sup>[80]</sup> initiated the concept of various compactness of IFSTS, namely almost compact, nearly compact, etc and also studied intuitionistic soft fuzzy filter and intuitionistic soft fuzzy prime filter of IFSTS.

### 3.3 Neutrosophic soft set

This section contains the overview of various studies on NSS. In 2012, Maji<sup>[41]</sup> defined the concept of NSS by combining soft set and neutrosophic set. Some basic operations of NSS, such as union, intersection and complement are defined and developed some properties of NSS. In the year 2013, Said Broumi<sup>[20]</sup> presented the concept of generalized NSS with basic definitions and properties of generalized NSS. Deli<sup>[29]</sup> defined the notion of relation on NSS. The composition of NSS is used to compose two different NSS. Deli<sup>[29]</sup> examined the following concepts, namely equivalence relation, equivalence class and quotient of NSS. Deli also analyzed the decision making problem using NSS relation. Arockiarani<sup>[9]</sup> defined a distance measure and score function to present a decision making problem using the existing tool called NSS.

In 2017, Al-Quran<sup>[11]</sup> introduced the notion of neutrosophic vague soft set which is an extended concept of classical soft set. Some basic operations and properties are defined and studied and at the end of the work, presented the decision making problem using the proposed concept to show the effectiveness. Parimala et al.<sup>[56]</sup> introduced an algorithm to analyze the medical diagnosis problem using interval-valued FSTS. In their work, some basic theoretic operations are also investigated.

Bera<sup>[13]</sup> introduced the concept of NSTS. In the introduction paper, the authors are also defined interior, closure, base for NSTS, subspace of NSTS and regular NSTS. Finally some properties of NSTS and separation axioms with different characteristics are studied and investigated.

In 2018 Bera<sup>[17]</sup> introduced neutrosophic soft connected and compact topological space along with some properties. Finally the concept of continuous mapping on NSTS introduced and studied. Gunduz Aras et al.<sup>[52]</sup> established the definition of NSS and introduced the neutrosophic soft point. Finally separation axioms and subspace of NSTS are studied in detail. Parimala et al.<sup>[60]</sup> proposed a new concept by incorporating NSS with hesitancy degree, which is exclusively for finding the residual of NSS.

## 4 Advantages and Limitations

Advantages and limitation of classical topological space and other topological spaces, such as FSTS, IFSTS, NSTS are presented here.

Types	Advantages	Limitations
General topology	It's a classical method and it is basic for all other topological space.	We could not apply the classical approach for uncertainties and for many real life fields.
Fuzzy topology	In fuzzy topology, every element has membership grade which lies between $[0, 1]$ .	Rejection part of membership does not exist in the fuzzy topology.
Intuitionistic fuzzy topology	Every element in the set has truth and falsehood value.	It's difficult to apply when some element have indeterminacy or indeterminate form.
Neutrosophic topology	Every element in the non-empty set has acceptance, rejection and indeterminacy value. So all variables in the universe of discourse have value between $[0, 1]$ .	Residual part may lead to some obvious errors in the solution.
Fuzzy soft topology	A non-empty set can be written as disjoint union of parameters set. One can separate the characteristic from the universal set and investigate according to the need of problem.	Non-acceptance of an element in the parameter does not consider.
Intuitionistic fuzzy soft topology	Every element in the parameterization has possibility of acceptance and possibility of non-acceptance value.	Omitting the possibility of neutrality.
Neutrosophic soft topology	Here we consider acceptance rate, non-acceptance rate and neutrality rate of all elements in the parameter.	Accuracy may affect if the residual rate is high.

The following table emphasize the comparison of various tools which we discussed in this overview.

Sets	Image	Pre-Image	Uncertainty	Truth Value of Parameter	False value of Parameter	Indeterminacy of parameter.
Classical sets	Universal set	Integer Set.	-	-	-	-
Soft Topology	Initial Universe	Power set whose ranges from closed interval 0 to 1	-	-	-	-
Fuzzy Soft Topology	Initial Universe	Power set whose ranges from closed interval 0 to 1	Present	Present	-	-
Intuitionistic Soft Topological Space	Initial Universe	Power set whose ranges from closed interval 0 to 1	Present	Present	Present	-
Neutrosophic soft topological space	Initial Universe	Power set whose ranges from closed interval 0 to 1	Present	Present	Present	present

## 5 Conclusions

Topological space has several applications in mathematics and in other fields like operations research, physics, data science, etc. But sometimes applying the concept of topology for real life application is difficult, because of uncertainties, inconsistent, incomplete information of the element. Fuzzy soft topological space introduced to overcome the difficulty in classical set which deals uncertainty of the object and intuitionistic fuzzy soft topological space established to solve some problem which encounter in fuzzy soft topology. Some cases,

object has indeterminacy value, for those cases the previous tools can't be used. So Neutrosophic set has been introduced to deal the uncertainty, incomplete and inconsistent. This paper is thorough study of all these tools. Advantages and limitations of all existing tools are discussed.

## References

- [1] Abd El-Latif, A. M., Characterizations of fuzzy soft pre separation axioms, Journal of New Theory, 7, pp. 47-63, 2015.
- [2] Abd El-latif, A. M., Fuzzy soft  $\alpha$ -connectedness in fuzzy soft topological spaces, Math.Sci. Lett., 5 (1), pp. 85-91,2016.
- [3] Abd El-latif, A. M., Fuzzy soft separation axioms based on fuzzy  $\beta$ -open soft sets, Ann. Fuzzy Math. Inform., 11 (2), pp. 223-239,2016.
- [4] Abd El-latif, A. M., Hosny, R. A., Fuzzy soft pre-connected properties in fuzzy soft topological spaces, South Asian J Math, 5 (5), pp. 202-213, 2015.
- [5] Agarwal, M., Biswas, K. K. and Henmandlu, M., Generalised intuitionistic fuzzy soft sets with application in decision making, Applied Soft Computing, 13, pp. 3552-3566,2013.
- [6] Amit Kumar Singh., Rekha Srivastava, Separation axioms in intuitionistic fuzzy topological spaces, Hindawi Publishing Corporation Advances in Fuzzy Systems , Article ID 604396, pp. 1-7,2012.
- [7] Arockiarani. I., Martina Jency. J., More on fuzzy neutrosophic sets and fuzzy neutrosophic topological spaces, Inter.J. Innov. Research & Studies, 3 (5), 643-652,2014.
- [8] Arockiarani, I., Sumathi, I. R. and Martina Jency, J., Fuzzy neutrosophic soft topological spaces, Inter.J.Math.Archive, 4 (10), pp. 225-238,2013.
- [9] Arockiarani. I., A Fuzzy neutrosophic soft set model in medical diagnosis, in:IEEE conference on Norbert Wiener in the 21st Century, (2014), Boston, MA, USA.
- [10] Aygunoglu, A., Cetkin, V., Aygun, H., An introduction to fuzzy soft topological spaces, Hacettepe Journal of Mathematics and Statistics, 43 (2) ,pp. 197 – 208, 2014.
- [11] Al-Quran, A., Hassan, N., Neutrosophic vague soft set and its application, Malaysian Journal of Mathematical Science, 11 (2), pp. 141-163,2017.
- [12] Atanassov, K., Intuitionistic fuzzy sets, Fuzzy Sets and Syst., 20, pp. 87-96,1986.
- [13] Bayramov, S., Gunduz, C., On intuitionistic fuzzy soft topological spaces, TWMS. J. Pure Appl. Math., 5, pp. 66-79,2014.
- [14] Bayramov, S., Gunduz, C., A New approach to separability and compactness in soft topological spaces. TWMS J. Pure Appl. Math., 9, pp. 82-93,2018.
- [15] Bayramov,S. and Gunduz, G., On intuitionistic fuzzy soft topological spaces, Inter.J.PureAppl.Math, 5(1), pp. 66-79,2014.
- [16] Bora,M., Neog, T. J., Sut,D.K., Some new operations of intuitionistic fuzzy soft sets, International Journals of Soft Computing and Engineering, 2(4), pp.2231-2307,2012.
- [17] Bera, T., Mahapatra, N. K., On neutrosophic soft topological space, Neutrosophic Sets and Systems, 19, pp. 3- 15,2018.
- [18] Bera, T., Mahapatra, N.K., Introduction to neutrosophic soft topological space, OPSEARCH, 54(4),pp. 841-867,2017.
- [19] Broumi, S., and Smarandache, F., Intuitionistic neutrosophic soft set, Journal of Information and Computing Science, 8 (2), pp. 130-140,2013.
- [20] Broumi, S., Generalized neutrosophic soft set, IJCSEIT,3 (2), pp. 17-30,2013.

- [21] Cagman, N., Karatas, S., Enginoglu, S., Soft topology, Computer and Mathematics with Application, 62, pp. 351-358,2011.
- [22] Cagman, N., Citak, F. and Enginoglu, S., Fuzzy parameterized fuzzy soft set theory and its applications, Turkish Journal of Fuzzy Systems, 1 (1), pp. 21-35,2010.
- [23] Cagman, N., Enginoglu, S., Citak,F., Fuzzy soft set theory and its application, Iranian Journal of fuzzy systems,8 (3), pp. 137-147,2011.
- [24] Chang, C. L., Fuzzy topological spaces, J. Math. Anal. Appl., 24, pp.182–190,1968.
- [25] Cetkin, V., Aygun, H., A Note on fuzzy soft topological spaces, Conference of the European Society for Fuzzy Logic and Technology,2013.
- [26] Cetkin, V., Aygun, H., A Note on intuitionistic supra fuzzy soft topological spaces, Notes on Intuitionistic Fuzzy Sets, 21 (4), pp. 48-57,2015.
- [27] Coker, D., An Introduction to intuitionistic fuzzy topological space, Fuzzy Sets and Systems, 88, pp. 81-89,1997.
- [28] Deli, I., npn-Soft sets theory and applications, Annals of Fuzzy Mathematics and Informatics, 10 (6), pp. 847 -862,2015.
- [29] Deli, I., Broumi, S., Neutrosophic soft relations and some properties, Annals of Fuzzy Mathematics and Informatics, 9 (1), pp. 169-182,2015.
- [30] Dey, A., Pal, M., Generalised multi-fuzzy soft set and its application in decision making, Pacific Science Review A: Natural Science and Engineering, 17, pp. 23-28,2015.
- [31] Gunduz Aras, C., Bayramov, S., Some results on fuzzy soft topological spaces, Math. Prob. in Engg., 20, pp. 1-10,2013.
- [32] Gunduz Aras, C., Bayramov, S., Separation axioms on neutrosophic soft topological spaces, Turk J Math, 43, pp. 498 – 510,2019.
- [33] Jiang, J., Tang,Y., Chen,Q., Li, H., Tang,J., Interval-valued intuitionistic fuzzy soft sets and their properties, Computer and Mathematics with Application, 60, pp. 906-918,2010.
- [34] Lu,J., Pan,L., Yang,Y., A Group medical diagnosis model based on intuitionistic fuzzy soft sets, Applied soft computing Journal, 77, pp. 453-466,2017.
- [35] Kalpana, G., Kalaivani, C., Fuzzy soft topology, International Journal of Engineering Studies,9 (1), pp. 45-56,2017.
- [36] Kandil, A., Tantawy,O. A. E., El-Sheikh, S. A., Abd El-latif, A. M. and El-Sayed. S., Fuzzy soft  $\alpha$ -connectedness in fuzzy soft topological spaces, Journal of Mathematics and Computer Applications Research, 2 (1), pp. 37-46,2015.
- [37] Kandil, A., Tantawy,O. A. E., El-Sheikh, S. A., Abd El-latif, A. M., Fuzzy soft semi connected properties in fuzzy soft topological spaces, Math. Sci. Lett., 4, pp. 171-179,2015.
- [38] Kumud Borgohain., Separation axioms in intuitionistic fuzzy soft topological space, International Journal of Mathematics Trends and Technology, 5, pp. 176-180,2014.
- [39] Lowen, R., Fuzzy topological spaces and fuzzy compactness, J. Math. Anal. Appl. 56, pp. 621–633,1976.
- [40] Li, Z., Cui, R., On the topological structure of intuitionistic fuzzy soft sets, Annals of Fuzzy Mathematics and Informatics, 5 (1), pp. 229-239,2013.
- [41] Maji, P. K., Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, 5, pp. 157-168,2013.
- [42] Maji,P.K., Biswas, R., Roy, A. R., Fuzzy soft sets, Journal of Fuzzy Mathematics, 9 (3), pp. 589-602,2001.
- [43] Maji,P.K., Biswas,R., Roy, A. R., Intuitionistic fuzzy soft sets, Journal of Fuzzy Mathematics, 9 (3), pp. 677-691,2001.

- [44] Maji,P.K., An application of weighted neutrosophic soft sets in a decision making problem, Springer proceedings in Mathematics and Statistics, 125, pp. 215-223,2015.
- [45] Mahanta, J., Das, P. K., Results on fuzzy soft topological spaces, arXiv:1203.0634v1,2012.
- [46] Majumdar, P., Samanta, S.K., Generalised fuzzy soft sets, Computers and Mathematics with Application, 59, pp. 1425-1432,2010.
- [47] Molodtsov, D., Soft set theory-first results, Computers and Mathematics with Application, 37, pp. 19-31,1999.
- [48] Mahanta, J., Das, P. K., Fuzzy soft topological spaces, Journal of Intelligent & Fuzzy Systems, 32, pp.443–450,2017.
- [49] Mukherjee, A., Das,A. K., Saha, A., Sequence in intuitionistic fuzzy soft multi topological spaces, NTM-SCI, 3( 3), pp. 184-191,2015 .
- [50] Mukherjee, A., Das,A. K., Topological structure formed by fuzzy soft multi sets, Bulletin of Cal. Math. Soc., 21, pp. 193-212,2013.
- [51] Mukherjee, A., Das,A. K., Parameterized topological space induced by an intuitionistic fuzzy soft multi topological space, Annals of Pure and Applied Math., 7 pp. 7-12,2014.
- [52] Mukherjee, A., Das, A. K. and Saha, A., Topological structure formed by soft multi sets and soft multi compact space, Annals of Fuzzy Mathematics and Informatics, 7, pp. 919-933,2014.
- [53] Muthukumar,P., Sai sundara Krishnan, G., A similarity measure of intuitionistic fuzzy soft sets and its application in medical diagnosis, Applied Soft Computing, 41, pp. 148- 156,2016.
- [54] Neog, T., Su, D. K., Hazarika, G. C., Fuzzy soft topological spaces, Inter J. Latest Trend Math., 2(1), pp. 87-96,2012.
- [55] Osmanoglu, I., Tokat, D., On intuitionistic fuzzy soft topology ,Gen. Math. Notes, 19 (2), pp. 59-70,2013.
- [56] Parimala, M., Karthika, M., Jafari, S., Smarandache, F. and Udhayakumar, R., Decision- making via neutrosophic support soft topological spaces, Symmetry, 217(10), pp. 1-10,2018.
- [57] Parimala, M., Karthika, M., Dhavaseelan, R. and Jafari, S., On neutrosophic supra pre- continuous functions in neutrosophic topological spaces, New Trends in Neutrosophic Theory and Applications, 2, pp. 371-383,2017.
- [58] Parimala, M., Indirani, C., Perumal, R., On intuitionistic fuzzy contra psi-continuous mappings in topological spaces, International Journal of Pure and Applied Mathematics, 113 (12), pp. 107-114,2017.
- [59] Parimala, M., Indirani, C., On intuitionistic fuzzy beta-supra open set and intuitionistic fuzzy beta-supra continuous functions, Notes on Intuitionistic Fuzzy Sets, 20 (3), pp. 6-12,2014.
- [60] Parimala, M., Karthika, M., Smarandache, F., Broumi,S., On  $\alpha\omega$ -closed sets and its connectedness in terms of neutrosophic topological spaces, International Journal of Neutrosophic Science, 2(2), PP: 82-88 , 2020.
- [61] Roy, S., Samanta, T. K., A Note on fuzzy soft topological spaces, Annals of fuzzy mathematics and informatics, 3(2), pp. 305-311,2012.
- [62] Roy, A. R., Maji, P. K., A Fuzzy soft set theoretic approach to decision making problems, Journal of Computational and Applied Mathematics, 203, pp. 412–418,2007.
- [63] Roy, S., Samanta, T. K., An introduction to open and closed sets on fuzzy soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 6 (2), pp. 425- 431,2012.
- [64] Sabir Hussain., On weak and strong forms of fuzzy soft open sets, Fuzzy information and Engineering, 8 (4), pp. 451-463,2016.
- [65] Salama, A.A., Alblowi, S.A.,Neutrosophic set and neutrosophic topological spaces, IOSRJ.Mathematics, 3 (4), pp. 31-35,2012.

- [66] Sahin, M., Alkhazaleh, S., Ulucay, V., Neutrosophic soft expert sets, *Applied Mathematics*, 6, pp. 116-127, 2015.
- [67] Simsekler, T., Yuksel, S., Fuzzy soft topological space, *Annals of fuzzy mathematics and Informatics*, 5 (1), pp. 87-96, 2013.
- [68] Smarandache, F., Neutrosophy and neutrosophic logic, *First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics*, University of New Mexico, Gallup, NM 87301, USA (2002).
- [69] Smarandache, F., Neutrosophic set, a generalization of the intuitionistic fuzzy sets, *Inter.J.Pure Appl.Math.*, 24, pp. 287 – 297, 2005.
- [70] Shabir, M., Naz, M., On soft topological spaces, *Comput. Math. Appl.*, 61, pp. 1786-1799, 2011.
- [71] Shuker Mahmood Khalil., On intuitionistic fuzzy soft  $\beta$ -closed sets in intuitionistic fuzzy soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 10 (2), pp. 221-233, 2015.
- [72] Tanay, B., Burc Kandemir, M., Topological structure of fuzzy soft sets, *Comp. Math. Appl.*, 61, pp. 2952-2957, 2011.
- [73] Varol, B. P., Aygun, H., Fuzzy soft topology, *Hacettepe Journal of Mathematics and Statistics*, 41 (3), pp. 407 – 419, 2012.
- [74] Xu, Y., Sun, Y., Li, D., Intuitionistic fuzzy soft set, *2nd International workshop on intelligent systems and applications*, IEEE, China, 2010.
- [75] Yang, C., A note on soft set theory, *Computers and Mathematics with Applications*, 56, pp. 1899–1900, 2008.
- [76] Yang, X., Lin, T. S., Yang, J., Li, Y. and Yu, D., Combination of interval-valued fuzzy set and soft set, *Computers and Mathematics with Applications* 58, pp. 521–527, 2009.
- [77] Yang, Y., Tan, X. and Meng, C., The multi-fuzzy soft set and its application in decision making, *Applied Mathematical Modelling*, 37, pp. 4915-4923, 2013.
- [78] Yao, B. X., Liu, J. L., Yan, R. X., Fuzzy soft set and soft fuzzy set, *Fourth international conference on natural computation*, IEEE, 2008.
- [79] Yin, Y., Li, H. and Jun, Y. B., On algebraic structure of intuitionistic fuzzy soft sets, *Computer and Mathematics with Applications*, 64, pp. 2896-2911, 2012.
- [80] Yogalakshmi, T., On intuitionistic soft fuzzy compactness, *International Journal of Pure and Applied Mathematics*, 115 (9), pp. 231-237, 2017.
- [81] Zadeh, L.A., Fuzzy sets, *Information and Control*, 8, pp. 338–353, 1965.
- [82] Zhi Kong., Lifu Wang., Zhaoxia Wu., Application of fuzzy soft set in decision making problems based on grey theory, *journal of Computational and applied Mathematics*, 236, pp. 1521-1530, 2011.



# Interval-Valued Triangular Neutrosophic Linear Programming Problem

Bhimraj Basumatary<sup>1,\*</sup> and Said Broumi<sup>2</sup>

<sup>1</sup>Department of Mathematical Sciences, Bodoland University Kokrajhar, India; [brbasumatary14@gmail.com](mailto:brbasumatary14@gmail.com)

<sup>2</sup>Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, Casablanca, Morocco; [broumisaid78@gmail.com](mailto:broumisaid78@gmail.com)

\* Correspondence: [brbasumatary14@gmail.com](mailto:brbasumatary14@gmail.com)

## Abstract

In this paper, we have proposed an Interval-valued triangular neutrosophic number (IV-TNN) as a key factor to solve the neutrosophic linear programming problem. In the present neutrosophic linear programming problem IV-TNN is expressed in lower, upper truth membership function, indeterminacy membership function, and falsity membership function. Here, we try to compare our proposed method with existing methods.

**Keywords:** Neutrosophic Set, Interval-valued triangular neutrosophic number, Neutrosophic Linear Programming Problem.

## 1.Introduction

A fuzzy set is defined by Zadeh[1] in the year 1965. Application of fuzzy set theory in various fields like decision making particularly in Linear programming problems (optimization problems) have been studied in a wide area after the introduction of a fuzzy set. As a result, many researchers have a contribution in this direction. Zadeh's [1] definition on fuzzy set theory, it shows because of uncertainty for determining the degree of the membership function is not possible. In this regard, Zadeh[2] tried to sort out the problem and in the year 1975, he proposed an interval-valued fuzzy set. After the discovery of the fuzzy set theory, Atanassov[3] introduced the concept of the Intuitionistic Fuzzy set which is an extension of the fuzzy set. The intuitionistic fuzzy set explains the degree of non-membership denoting the not belongs to the set. The important extension of the fuzzy set is the interval-valued fuzzy set, which is characterized by an interval-valued membership function. In [4], Atanassov discussed an idea on an interval-valued intuitionistic fuzzy set. D. Dubey [5] proposed an approach based on value and ambiguity indices to solve LPPs with data as Triangular Intuitionistic Fuzzy Numbers.

Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information. By observing this in 1995, Smarandache [6-9] introduce neutrosophy, which is the study of neutralities as an extension of dialectics. Neutrosophic sets are characterized by three independent degrees namely truth- membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F), where T,I,F are standard or non-standard subsets of  $]0^-,1^+[$ . The definition of single-valued neutrosophic sets were introduced by Wang [10], whose values belong to  $[0,1]$ . Single-valued neutrosophic sets were successfully applied to various decision-making problems [11–16].

Linear programming is most powerful technique, which occurs in decision-making. Bellman and Zadeh [17] introduced fuzzy optimization problems where they have stated that a fuzzy decision can be viewed as the intersection of fuzzy goals and problem constraints. Many researchers such as; Zimmermann [18], Tanaka, et al.[19], Campos and Verdegay [20], Rommelfanger et al.[21], Cadenas and Verdegay [22] who were dealing with the concept of solving fuzzy optimization problems, later studied this subject. Parvathi and Malathi [23] have done their work on intuitionist fuzzy linear optimization.

Then Abdel-Baset et al.[24] and pramanik [25] proposed neutrosophic linear programming methods based on the idea of neutrosophic sets. Also, Abdel-Baset et al.[26] introduced the neutrosophic linear programming models where their parameters are represented with the trapezoidal neutrosophic numbers and presented a technique for solving problems. Nafei et al. [27] presented a new method for solving interval neutrosophic linear programming problems. Abdel-Baset et al.[25] discussed a novel method for solving the fully neutrosophic linear programming problems. Khatter [39] proposed Neutrosophic linear programming using possibilistic mean according to Kirna[39] the proposed approach converts each triangular neutrosophic number in linear programming problem to weighted value using possibilistic mean to determine the crisp linear programming problem. Bera et al.[40] approach the application of neutrosophic linear programming problem to real life. They proposed an algorithm of the Big-M simplex method in this new climate and then it is applied to a real-life problem. Ye [41] studied on neutrosophic number linear programming method and its application under neutrosophic number environments.

In this paper, we proposed an interval-valued triangular neutrosophic number to solve the neutrosophic linear programming problem so that we could have a better result in comparison to other methods. The structure of the chapter is as follows: the next section is a preliminary discussion; the third section describes the interval-valued triangular neutrosophic number of the proposed model; the fourth section describes steps of the proposed model; the last section summaries the conclusion.

## 2. Preliminaries

**Definition 2.1 [6]** Let  $X$  be a universe of discourse. A single-valued neutrosophic set  $N$  through  $X$  taking the form  $N = \{x, T_N(x), I_N(x), F_N(x) : x \in X\}$ , where  $T_N(x) : X \rightarrow [0, 1]$ ,  $I_N(x) : X \rightarrow [0, 1]$  and  $F_N(x) : X \rightarrow [0, 1]$  with  $0 \leq T_N(x) \leq I_N(x) \leq F_N(x) \leq 3$  for all  $x \in X$  and  $T_N(x), I_N(x)$  and  $F_N(x)$  represents truth membership function, indeterminacy membership function, and falsity membership function of  $x$  to  $N$ .

**Definition:2.2[28]** A triangular neutrosophic number (TNN) is denoted by  $A^N = \{(a^l, a^m, a^u), (\mu, i, \gamma)\}$  whose the three membership functions for the truth membership function, indeterminacy membership function and falsity membership function of  $x$  can be defined as

$$T_{A^N}(x) = \begin{cases} \frac{(x - a^l)}{(a^m - a^l)} \mu, & a^l \leq x \leq a^m \\ \mu, & x = a^m \\ \frac{(a^u - x)}{(a^u - a^m)} \mu, & a^m \leq x \leq a^u \\ 0, & \text{otherwise} \end{cases}$$

$$I_{A^N}(x) = \begin{cases} \frac{(x - a^l)}{(a^m - a^l)} i, & a^l \leq x \leq a^m \\ i, & x = a^m \\ \frac{(a^u - x)}{(a^u - a^m)} i, & a^m \leq x \leq a^u \\ 0, & \text{otherwise} \end{cases}$$

$$F_{A^N}(x) = \begin{cases} \frac{(x - a^l)}{(a^m - a^l)} \gamma, & a^l \leq x \leq a^m \\ \gamma, & x = a^m \\ \frac{(a^u - x)}{(a^u - a^m)} \gamma, & a^m \leq x \leq a^u \\ 0, & \text{otherwise} \end{cases}$$

Where  $0 \leq T_{A^N}(x) + I_{A^N}(x) + F_{A^N}(x) \leq 3$ , for  $x \in A^N$  and if  $a^l \geq 0$ ,  $A^N$  is called a nonnegative TNN and for  $a^l \leq 0$ ,  $A^N$  is called a negative TNN.

**Definition 2.3 [28]** Let  $\tilde{A} = \{(a_1, a_2, a_3), (\mu_1, i_1, \gamma_1)\}$  and  $\tilde{B} = \{(b_1, b_2, b_3), (\mu_2, i_2, \gamma_2)\}$  be two TNN then the mathematical operation on two triangular neutrosophic numbers  $\tilde{A}$  and  $\tilde{B}$  are as follows

$$\tilde{A} + \tilde{B} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3), (\mu_1 \wedge \mu_2, i_1 \vee i_2, \gamma_1 \vee \gamma_2)\}$$

$$\tilde{A} - \tilde{B} = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1), (\mu_1 \vee \mu_2, i_1 \wedge i_2, \gamma_1 \wedge \gamma_2)\}$$

$$\lambda \tilde{A} = \begin{cases} \{(\lambda a_1, \lambda a_2, \lambda a_3), (\mu_1, i_1, \gamma_1)\} & \text{if } \lambda \geq 0 \\ \{(\lambda a_3, \lambda a_2, \lambda a_1), (\mu_1, i_1, \gamma_1)\} & \text{if } \lambda \leq 0 \end{cases}$$

### 3. Interval Valued Triangular Neutrosophic Number (IV-TNN)

**Definition 3.1** Interval-valued triangular neutrosophic number (IV-TNN) shows a more abundant and flexible result than the neutrosophic number. Many authors defined with a different type of triangular neutrosophic numbers in this work we will define interval-valued triangular neutrosophic number (IV-TNN).

An IV-TNN is defined as  $\tilde{A} = \{(a^L, a, c^L)(a^U, a, c^U)[\mu^L, \mu^U]; (e^L, a, g^L)(e^U, a, g^U)[I^L, I^U]; (l^L, a, n^L)(l^U, a, n^U)[\gamma^L, \gamma^U]\}$ , where  $a^L, a, c^L, a^U, c^U, e^L, g^L, e^U, g^U, l^L, n^L, l^U, n^U$  are belong to  $\mathbf{R}$  and its lower, upper truth membership function, indeterminacy membership function, and falsity membership function is defined as follows

(a). Lower and upper truth membership functions are

$$\tau_{\tilde{A}}^L(x) = \begin{cases} \frac{(x - a^L)}{(a - a^L)} \mu^L, & a^L \leq x \leq a \\ \mu^L, & x = a \\ \frac{(c^L - x)}{(c^L - a)} \mu^L, & a \leq x \leq c^L \\ 0, & \text{otherwise} \end{cases}, \quad \tau_{\tilde{A}}^U(x) = \begin{cases} \frac{(x - a^U)}{(a - a^U)} \mu^U, & a^U \leq x \leq a \\ \mu^U, & x = a \\ \frac{(c^U - x)}{(c^U - a)} \mu^U, & a \leq x \leq c^U \end{cases}$$

(b). Lower and upper indeterminacy membership functions are

$$I_{\tilde{A}}^L(x) = \begin{cases} \frac{(x-e^L)}{(a-e^L)} i^L, & e^L \leq x \leq a \\ i^L, & x = a \\ \frac{(g^L-x)}{(g^L-a)} i^L, & a \leq x \leq g^L \\ 0, & \text{otherwise} \end{cases}, \quad I_{\tilde{A}}^U(x) = \begin{cases} \frac{(x-e^U)}{(a-a^U)} i^U, & a^U \leq x \leq a \\ i^U, & x = a \\ \frac{(g^U-x)}{(g^U-a)} i^U, & a \leq x \leq g^U \end{cases}$$

(c) Lower and upper falsity membership functions are

$$F_{\tilde{A}}^L(x) = \begin{cases} \frac{(x-l^L)}{(a-l^L)} \gamma^L, & l^L \leq x \leq a \\ \gamma^L, & x = a \\ \frac{(n^L-x)}{(n^L-a)} \gamma^L, & a \leq x \leq n^L \\ 0, & \text{otherwise} \end{cases}, \quad F_{\tilde{A}}^U(x) = \begin{cases} \frac{(x-l^U)}{(a-l^U)} \gamma^U, & l^U \leq x \leq a \\ \gamma^U, & x = a \\ \frac{(n^U-x)}{(n^U-a)} \gamma^U, & a \leq x \leq n^U \end{cases}$$

**Remark:1.** If  $a^L = a^U = e^L = e^U = l^L = l^U = c^L = c^U = g^L = g^U = n^L = n^U$  then IV-TNN becomes a triangular neutrosophic number.

**Remark:2.** If  $a^L = a^U = e^L = e^U = l^L = l^U = c^L = c^U = g^L = g^U = n^L = n^U$  and  $\mu^L = \mu^U, l^L = l^U$  and  $\gamma^L = \gamma^U$  then IV-TNN becomes neutrosophic set

### Definition 3.2

Let

$\tilde{M} = \{(a_M^L, a, c_M^L)(a_M^U, a, c_M^U)[\mu_M^L, \mu_M^U]; (e_M^L, a, g_M^L)(e_M^U, a, g_M^U)[I_M^L, I_M^U]; (l_M^L, a, n_M^L)(l_M^U, a, n_M^U)[\gamma_M^L, \gamma_M^U]\}$  and

$\tilde{N} = \{(a_N^L, a, c_N^L)(a_N^U, a, c_N^U)[\mu_N^L, \mu_N^U]; (e_N^L, a, g_N^L)(e_N^U, a, g_N^U)[I_N^L, I_N^U]; (l_N^L, a, n_N^L)(l_N^U, a, n_N^U)[\gamma_N^L, \gamma_N^U]\}$  then

$$\begin{aligned} \tilde{M} \oplus \tilde{N} = & \left\{ (a_M^L + a_N^L, 2a, c_M^L + c_N^L) (a_M^U + a_N^U, 2a, c_M^U + c_N^U) [\mu_M^L + \mu_N^L - \mu_M^L \mu_N^L, \mu_M^U + \mu_N^U - \mu_M^U \mu_N^U]; \right. \\ & (e_M^L + e_N^L, 2a, g_M^L + g_N^L) (e_M^U + e_N^U, 2a, g_M^U + g_N^U) [I_M^L I_N^L, I_M^U I_N^U]; \\ & (l_M^L + l_N^L, 2a, n_M^L + n_N^L) (l_M^U + l_N^U, 2a, n_M^U + n_N^U) [\gamma_M^L + \gamma_N^L, \gamma_M^U + \gamma_N^U] \left. \right\} \end{aligned}$$

$k\tilde{M}$

$$= \begin{cases} \{(ka_M^L, ka, kc_M^L)(ka_M^U, ka, kc_M^U)[\mu_M^L, \mu_M^U]; (ke_M^L, ka, kg_M^L)(ke_M^U, ka, kg_M^U)[I_M^L, I_M^U]; (kl_M^L, ka, kn_M^L)(kl_M^U, ka, kn_M^U)[\gamma_M^L, \gamma_M^U]\}; & \text{if } k > 0 \\ \{(ka_M^U, ka, kc_M^U)(ka_M^L, ka, kc_M^L)[\mu_M^L, \mu_M^U]; (ke_M^U, ka, kg_M^U)(ke_M^L, ka, kg_M^L)[I_M^L, I_M^U]; (kl_M^U, ka, kn_M^U)(kl_M^L, ka, kn_M^L)[\gamma_M^L, \gamma_M^U]\}; & \text{if } k < 0 \\ 0 & \text{if } k = 0 \end{cases}$$

### 3.3. Neutrosophic Linear Programming Problem (NLPP)

Linear programming is an optimization technique widely used in practical problem. In this section we generalize the LPP term as the interval-valued triangular neutrosophic programming problem, denoted as IV-TNLP problem and defined as

Maximize/minimize  $\tilde{Z} = \tilde{c} x$

Such that  $\tilde{A} x \leq \tilde{b}, x \geq 0$

Where  $\tilde{c}, \tilde{A}, \tilde{b}$  are interval-valued triangular neutrosophic numbers.

### 4. Proposed IV-TNLP Method

**Step1** Let the decision-makers insert their IV-TNLP problem. Because we always try to maximize truth membership function and minimize indeterminacy membership function and falsity membership, then inform decision-makers to apply the concept when entering triangular neutrosophic numbers of the IV-TNLP problem.

**Step 2** Convert IV-TNLP problem to its crisp model by using the following method

Let  $\tilde{A} = \{(a^L, a, c^L)(a^U, a, c^U), [\mu^L, \mu^U]; (e^L, a, g^L)(e^U, a, g^U), [l^L, l^U]; (l^L, a, n^L)(l^U, a, n^U), [\gamma^L, \gamma^U]\}$  be an interval-valued triangular neutrosophic number where,  $[\mu^L, \mu^U]$ ,  $[l^L, l^U]$  and  $[\gamma^L, \gamma^U]$  are truth membership function, indeterminacy membership function and falsity membership function of  $\tilde{A}$ . The ranking function for an interval-valued neutrosophic number  $\tilde{A}$  will be defined as

$$R(\tilde{A}) = \frac{a^L + a^U + c^L + c^U + e^L + e^U + 12a + g^L + g^U + l^L + l^U + n^L + n^U}{24} + \frac{\mu^L + \mu^U}{2} - \frac{l^L + l^U}{2} - \frac{\gamma^L + \gamma^U}{2}.$$

**Step 3** By applying the proposed ranking function converts each interval-valued triangular neutrosophic number to its equivalent crisp value. This leads to convert the IV-TNLP problem to its crisp model.

**Step 4** Solve the crisp model using the standard method and obtained the optimal solution to the problem.

### 5 Comparison of the proposed method with existing methods

#### Comparison of the proposed method with [23]

Maximize  $\tilde{Z} = \tilde{3} x_1 + \tilde{4} x_2$

Such that,

$$\tilde{1} x_1 + \tilde{2} x_2 \leq \tilde{20},$$

$$\tilde{3} x_1 + \tilde{5} x_2 \leq \tilde{60}$$

$$x_1, x_2 \geq 0$$

According to [23] the optimal value of the objective function is  $Z=21.45$ . Now we assume the parameters of LPP according to the proposed method which are represented as follows –

$$\tilde{3} = \{(2.55, 3, 6.55)(2.65, 3, 6) [0.40, 0.60]; (2.25, 3, 4.80)(2.65, 3, 6.80) \\ [0.1, 0.2]; (2.75, 3, 6.50)(2.80, 3, 7.85) [0.3, 0.5]\}$$

$$\tilde{4} = \{(2.5, 4, 5.9)(2.9, 4, 6)[0.3, 0.5]; (2.1, 4, 5.6)(2.4, 4, 5.8)[0.2, 0.3]; \\ (3, 4, 4.5), (3.5, 4, 5)[0.7, 0.9]\}$$

$$\tilde{1} = \{(0.2, 1, 1.9)(0.3, 1, 2)[0.8, 0.9]; (0.1, 1, 1.2)(0.2, 1, 1.3)[0.1, 0.3]; \\ (0.4, 1, 1.2)(0.8, 1, 1.8)[0.3, 0.4]\}$$

$$\tilde{2} = \{(1.4, 2, 2.4)(1.7, 2, 3)[0.3, 0.4]; (1.1, 2, 2.5)(1.5, 2, 2.6)[0.1, 0.3]; \\ (1.8, 2, 2.1)(1.9, 2, 2.9)[0.1, 0.3]\}$$

$$\tilde{3} = \{(2.6, 3, 3.9)(2.7, 3, 4)[0.3, 0.4]; (2.4, 3, 3.2)(2.5, 3, 3.5)[0.2, 0.3]; \\ (2.8, 3, 3.1)(2.9, 3, 3.9)[0.5, 0.6]\}$$

$$\tilde{5} = \{(4.3, 5, 6.1)(4.4, 5, 6.3)[0.2, 0.6]; (4, 5, 6)(4.2, 5, 4.8)[0.4, 0.6]; \\ (4.5, 5, 5.6)(4.8, 5, 6)[0.3, 0.4]\}$$

$$\tilde{20} = \{(19.1, 20, 20.4) (19.2, 20, 20.6) [0.4, 0.6]; (19, 20, 20.2) (19.5, 20, 20.5) \\ [0.1, 0.2]; (19.5, 20, 20.1)(19.9, 20, 20.3)[0.2, 0.6]\}$$

$$\tilde{60} = \{(59.50, 60, 71.80)(58.50, 60, 72.50)[0.5, 0.6]; (59, 60, 61.50) \\ (59.85, 60, 62.50)[0.2, 0.3]; (59.20, 60, 62.85)(59.70, 60, 70.50)[0.3, 0.4]\}$$

By using proposed ranking function the problem will be converted to the crisp model as follows

$$\text{Maximize } \tilde{Z} = 3.77 x_1 + 3.95 x_2$$

Such that,

$$0.78 x_1 + 1.96 x_2 \leq 19.88,$$

$$2.51 x_1 + 4.59 x_2 \leq 61.51$$

$$x_1, x_2 \geq 0$$

Solving the problem the optimal solution is  $x_1 = 24.51$ ;  $x_2 = 0$  with optimal objective value 92.39.

By comparing the proposed model results with the results of [23] of the same problem, we observed that our proposed model results are better than the results of [23]. Also, if we see the optimal solution of the existing solution under the intuitionistic fuzzy system are  $x_1 = 7.15$ ,  $x_2 = 0$  and  $Z_{[23]} = 21.45$  and from the optimal solution of the proposed method linear programming problem the objective function value equals 92.39 which is a problem of maximization. The proposed approach is smoother than the existing method in [23]. The existing method in [23] is only able to solve the problem but we can handle the situation in interval-valued neutrosophic number and due to this we can convert each interval-valued triangular neutrosophic number to its equivalent crisp value in a better way in comparison to

existing method. In addition, due to the explanation of determining truth membership function, falsity membership function, and indeterminacy-membership function, the proposed model is more effective than their method in [23].

### Comparison of the proposed method with [27]

$$\text{Maximize } \tilde{Z} = \tilde{7} x_1 + \tilde{6} x_2 + \tilde{14} x_3$$

Such that ,

$$\tilde{15} x_1 + \tilde{1} x_2 \leq \tilde{10},$$

$$\tilde{9} x_1 + \tilde{4} x_2 + \tilde{8} x_3 \leq \tilde{2}$$

$$\tilde{19} x_1 + \tilde{11} x_3 \leq \tilde{4}$$

$$x_1, x_2, x_3 \geq 0$$

Following the method [27], we can observe when  $a^L = a^U = e^L = e^U = l^L = l^U, c^L = c^U = g^L = g^U = n^L = n^U$  the proposed method and existing method in [27] shows the same result.

Now to get a more convenient optimal solution let us apply the proposed approach in comparison with the example of [27]. Let us assume IV-TNN as follows

$$\begin{aligned} \tilde{7} = & \{(5, 7, 9)(5.2, 7, 9.50)[0.70, 0.90]; (5.10, 7, 9.10)(5.50, 7, 9.50)[0.10, 0.40]; \\ & (5, 7, 8.50)(6.50, 7, 9)[0.20, 0.40]\} \end{aligned}$$

$$\begin{aligned} \tilde{6} = & \{(5.20, 6, 7.2)(5.8, 6, 7.8)[0.20, 0.60]; (5.30, 6, 6.85)(5.90, 6, 7)[0.20, 0.50]; \\ & (4.95, 6, 7.10), (5.50, 6, 7.25)[0.10, 0.80]\} \end{aligned}$$

$$\begin{aligned} \tilde{14} = & \{(10.55, 14, 17.50)(11.35, 14, 18.25)[0.50, 0.70]; (10.75, 14, 17.85) \\ & (11.80, 14, 18.55)[0.40, 0.60]; (11.95, 14, 17.55)(12.15, 14, 18.25)[0.30, 0.40]\} \end{aligned}$$

$$\begin{aligned} \tilde{15} = & \{(13.5, 15, 16.50)(14.10, 15, 16.85)[0.60, 0.80]; (14.20, 15, 17) \\ & (14.80, 15, 18.20)[0.10, 0.40]; (14.30, 15, 16.30)(14.55, 15, 17.4)[0.40, 0.90]\} \end{aligned}$$

$$\begin{aligned} \tilde{1} = & \{(0.85, 1, 1.95)(.90, 1, 2.90)[0.20, 0.70]; (0.55, 1, 2.15)(0.80, 1, 3.50) \\ & [0.10, 0.40]; (0.55, 1, 1.85)(0.80, 1, 2.85)[0.40, 0.95]\} \end{aligned}$$

$$\begin{aligned} \tilde{10} = & \{(5.85, 10, 16.25)(7.65, 10, 17.55)[0.1, 0.5]; (5.95, 10, 16.55) \\ & (7.55, 10, 17.85)[0.10, 0.40]; (6.80, 10, 16.85)(7.85, 10, 17.85)[0.60, 0.90]\} \end{aligned}$$

$$\begin{aligned} \tilde{9} = & \{(7.95, 9, 11.85)(8.55, 9, 13.55)[0.40, 0.50]; (8.55, 9, 11.85)(8.25, 9, 12.55) \\ & [0.50, 0.80]; (8.50, 9, 12.65)(8.35, 9, 14.85)[0.40, 0.80]\} \end{aligned}$$

$$\begin{aligned} \tilde{4} = & \{(3.55, 4, 6.95)(3.65, 4, 7.95)[0.10, 0.90]; (3.35, 4, 7.65)(3.65, 4, 8.85) \\ & [0.40, 0.5]; (3.25, 4, 6.85)(3.85, 4, 8.55)[0.30, 0.40]\} \end{aligned}$$

$$\begin{aligned}
\tilde{8} &= \{(6.85, 8, 9.5)(7.55, 8, 11.25)[0.70, 0.80]; (6.25, 8, 10.85)(7.25, 8, 11.55) \\
&\quad [0.50, 0.60]; (7.55, 8, 10.25)(7.85, 8, 11.25)[0.10, 0.60]\} \\
\tilde{2} &= \{(1.55, 2, 4.55)(1.75, 2, 5.55)[0.30, 0.60]; (1.45, 2, 4.85)(1.65, 2, 5.65) \\
&\quad [0.10, 0.90]; (1.85, 2, 4.45)(1.90, 2, 6.85)[0.40, 0.60]\} \\
\tilde{19} &= \{(16.85, 19, 21.80)(17.80, 19, 22.85)[0.50, 0.90]; (16.85, 19, 21.85) \\
&\quad (17.85, 19, 22.85)[0.30, 0.50]; (16.95, 19, 22.50)(18.25, 19, 23.55)[0.70, 0.80]\} \\
\tilde{11} &= \{(9.85, 11, 13.85)(10.55, 11, 14.85)[0.60, 0.80]; (9.25, 11, 13.85) \\
&\quad (10.85, 11, 14.85)[0.40, 0.90]; (10.20, 11, 14.55)(10.85, 11, 15.25)[0.60, 0.90]\} \\
\tilde{4} &= \{(2.55, 4, 5.55)(3, 4, 6.85)[0.30, 0.70]; (3, 4, 6.85)(3.85, 4, 7.25) \\
&\quad [0.10, 0.20]; (3.45, 4, 6.85)(3.85, 4, 7.25)[0.10, 0.30]\}
\end{aligned}$$

Now by using proposed ranking function the previous problem will be converted to the crisp model as follows

$$\text{Maximize } Z = 7.30 x_1 + 6 x_2 + 14.13 x_3$$

Such that,

$$15.01 x_1 + 0.30 x_2 \leq 10.26 ,$$

$$9.01 x_1 + 4.40 x_2 + 8.36 x_3 \leq 2.20$$

$$19.05 x_1 + 11.15 x_3 \leq 4.66$$

$$x_1, x_2, x_3 \geq 0$$

Solving the problem the optimal objective value is 3.72.

This clears that the proposed method shows a more convenient optimal value than the existing method in [27]. Though both the methods (existing and proposed) are the same when  $a^L = a^U = e^L = e^U = l^L = l^U, c^L = c^U = g^L = g^U = n^L = n^U$ , but due to having more chances for taking lower and upper triangular neutrosophic numbers for determining truth membership function, falsity membership function, and indeterminacy-membership function the proposed method shows the more suitable result. Hence our method is more applicable for solving real-life problems.

### Conclusion:

In this paper, we have tried to discuss the neutrosophic linear programming problem concerning the interval-valued triangular neutrosophic number and compare the proposed method with some papers. In the proposed method interval-valued triangular neutrosophic number is expressed in lower, upper truth membership function, indeterminacy membership function, and falsity membership function. It is seen that the proposed method shows better results in comparison to [23]. The second comparison with [27] shows almost the same result when  $a^L = a^U = e^L = e^U = l^L = l^U, c^L = c^U = g^L = g^U = n^L = n^U$  for the interval-valued triangular number that is when lower and upper truth membership function, indeterminacy membership function and falsity membership function, and interval-valued triangular number become the same. However, when we go through the proposed method it shows different in solution. In the future study, we extend the IV-TNLP algorithm in an interval-valued Neutrosophic Linear Fractional

Programming Problems. Hope our method would help in the future for getting better results in the decision-making system.

### **Acknowledgments**

The authors are very much thankful to the honorable referees for their valuable comments and suggestions for the improvement of the paper.

### **References**

- [1] L.A. Zadeh, Fuzzy Sets, Inform and Control 8, pp.338-353, 1965.
- [2] L.A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning, Information Sciences, 8, pp.199-249, 1975.
- [3] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Syst. 20, pp.87–96, 1986.
- [4] K.T. Atanassov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets Syst. 31, pp.343–349, 1989.
- [5] D. Dubey and A. Mehra, Linear Programming with Triangular Intuitionistic Fuzzy Number. European Society for Fuzzy Logic and Technology, pp.563-569, 2011. <https://doi.org/10.2991/eusflat.2011.78>
- [6] F.Smarandache, Neutrosophic set-a generalization of the intuitionistic fuzzy set, Inter- national Journal of Pure and Applied Mathematics, 24(3), 287–297, 2005.
- [7] I. M. Hezam, M. Abdel-Baset, F. Smarandache. Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem. In: Neutrosophic Sets and Systems. An International Journal in Information Science and Engineering, Vol. 10,, pp. 39-45, 2015.
- [8] N. El-Hefenawy, M. A. Metwally, Z. M. Ahmed, & El-Henawy, I. M. A Review on the Applications of Neutrosophic Sets. Journal of Computational and Theoretical Nanoscience, 13(1), pp. 936-944, 2016.
- [9] M. Abdel-Baset, I. M. Hezam & F. Smarandache, Neutrosophic Goal Programming. In: Neutrosophic Sets & Systems, vol. 11, 2016.
- [10] H. Wang, F. Smarandache, Y. Zhang, Sunderraman, R. Single valued neutrosophic sets. Multispace Multistruct. Neutrosophic Transdiscipl, 4, pp.410–413, 2010.
- [11] J. Ye, ; F. Smarandache, Similarity measure of refined single-valued neutrosophic sets and its multicriteria decision making method. Neutrosophic Sets Syst, 12, pp.41–44, 2016
- [12] J. Ye, Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. Int. J. Fuzzy Syst. 16, pp.204–211, 2014,
- [13] P. Liu, The aggregation operators based on archimedean t-conorm and t-norm for single-valued neutrosophic numbers and their application to decision making. Int. J. Fuzzy Syst., 18, pp.849–863, 2016.
- [14] J.Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. Int. J. Gen. Syst, 42, pp.386–394, 2013
- [15] Q. Hu, Zhang, X. New similarity measures of single-valued neutrosophic multisets based on the decomposition theorem and its application in medical diagnosis. Symmetry, 10, 466, 2018.
- [16] J. Wang, ; Zhang, X. Two types of single-valued neutrosophic covering rough sets and an application to decision making. Symmetry, 10, 710, 2018.

- [17] R.E. Bellman, L.A. Zadeh, Decision making in a fuzzy environment, *Manag. Sci.* 17, pp.141–164, 1970.
- [18] H.J. Zimmerman, Fuzzy programming and linear programming with several objective Functions, *Fuzzy Sets Syst.* 1, pp.45–55, 1978
- [19] H. Tanaka, K. Asai, A formulation of fuzzy linear programming based on comparison of fuzzy number, *Control and Cybernet.* 13, pp.185–194, 1984.
- [20] L. Campos, J.L. Verdegay, Linear programming problems and ranking of fuzzy numbers, *Fuzzy Sets Syst.* 32, pp.1–11, 1989.
- [21] H. Rommelfanger, R. Hanuscheck, J. Wolf, Linear programming with fuzzy objective, *Fuzzy Sets Syst.* 29, pp.31–48, 1998.
- [22] J. L. Verdegay, Using Fuzzy Numbers in Linear Programming, *System. Man. Cybernetics. Part B: Cybernetics. IEEE Transactions on* . 27, pp. 1016–1022, 1997.
- [23] R. Parvati, C. Malathi, Intuitionistic fuzzy linear optimization, *Notes on Intuitionistic Fuzzy Sets*, 18, pp.48–56, 2012.
- [24] S.K. Das, T. Mandal, & S.A. Edalatpanah, A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. *Applied intelligence*, 46(3), pp.509–519, 2017.
- [25] Abdel-Baset M, Hezam IM, Smarandache F., Neutrosophic goal programming, *Neutrosophic Sets Syst* 11, pp.112–118, 2016.
- [26] Pramanik, S., Neutrosophic multi-objective linear programming. *Global Journal of Engineering Science and Research Management*, 3(8), pp.36–46, 2016.
- [27] A. Nafei, M. Arif, W. Yuan and H. Nasseri, A new method for solving interval neutrosophic linear programming problems, *Easy Chair preprints (Acta Polytechnica Hungarica)*, No.- 2346, January 9, 2020
- [28] Abdel-Baset M, Gunasekaran M, Mai M, Smarandache F, A novel method for solving the fully neutrosophic linear programming problems, *Neural Computing and Application*, 2018, doi.org/10.1007/s00521-018-3404-6
- [29] Ye, J. (2014). Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. *Journal of Intelligent & Fuzzy Systems*, 26(1), pp.165–172.
- [30] Ji, P., Wang, J. Q., & Zhang, H. Y., Frank prioritized Bonferroni mean operator with singlevalued neutrosophic sets and its application in selecting third-party logistics providers. *Neural Computing and Applications*, 30(3), pp.799–823, 2018.
- [31] S. Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache, K. P. Krishnan Kishore, Ridvan Şahin, Shortest Path Problem under Interval Valued Neutrosophic Setting, *International Journal of Advanced Trends in Computer Science and Engineering*, Volume 8, No.1.1, pp.216–222, 2019.
- [32] S. Broumi, A. Dey, M. Talea, A. Bakali, F. Smarandache, D. Nagarajan, M. Lathamaheswari and Ranjan Kumar, “Shortest Path Problem using Bellman Algorithm under Neutrosophic Environment,” *Complex & Intelligent Systems*, pp.1–8, 2019. <https://doi.org/10.1007/s40747-019-0101-8>,
- [33] H. Tanaka, K. Asai, A formulation of fuzzy linear programming based on comparison of fuzzy number, *Control and Cybernet.* 13, pp.185–194, 1984.
- [34] L. Campos, J.L. Verdegay, Linear programming problems and ranking of fuzzy numbers, *Fuzzy Sets Syst.* 32, pp. 1–11, 1989.

- [35] Abdel-Basset M, Mohamed M, Zhou Y, Hezam I., Multi- criteria group decision making based on neutrosophic analytic hierarchy process. *J Intell Fuzzy Syst* 33(6), pp.4055–4066, 2017.
- [36] M. Mohamed, M. Abdel-Basset, A. N. Zaied, F. Smarandache, Neutrosophic integer programming problem, *Neutrosophic Sets Syst* 15, pp.3–7, 2017. <https://doi.org/10.5281/zenodo.570944>.
- [37] G. Nordo, A. Mehmood, Said Broumi; Single valued neutrosophic filter, *International Journal of Neutrosophic Science*, Vol.6, Issue-1, pp.8-21, 2020.
- [38] S. K. Das, S.A. Edalatpanah, A new ranking function of triangular neutrosophic number and its application in integer programming, *International Journal of Neutrosophic Science*, Vol.6, Issue-2, pp.82-92, 2020.
- [39] Khatter, K. Neutrosophic linear programming using possibilistic mean. *Soft Comput*, 2020. <https://doi.org/10.1007/s00500-020-04980-y>
- [40] Bera, T., Mahapatra, N.K. Neutrosophic linear programming problem and its application to real life. *Afr. Mat.* 31, pp.709–726, 2020. <https://doi.org/10.1007/s13370-019-00754-4>
- [41] Ye J., Neutrosophic number linear programming method and its application under neutrosophic number environments. *Soft Comput* 22(14), pp.4639–4646, 2018.



## **TOPSIS BY USING PLITHOGENIC SET IN COVID-19 DECISION MAKING**

<sup>1</sup>C. Sankar, <sup>2</sup>R. Sujatha, <sup>3</sup>D. Nagarajan

<sup>1</sup>Department of Mathematics, St. Joseph's College of Engineering,  
Sholinganallur, Chennai, India  
csankar26@gmail.com

<sup>2</sup>Department of Mathematics, Sri Sivasubramaniya Nadar College of Engineering,  
SSN-Centre for Radiation, Environmental Science and Radiation(SSN-CREST),  
Kalavakkam, Chennai, India  
sujathar@ssn.edu.in

<sup>3</sup>Department of Mathematics, Hindustan Institute of Technology and Science,  
Padur, Chennai, India  
drnagarajan75@gmail.com

### **Abstract**

COVID-19 is pandemic affecting most of the country globally. It is an infectious disease that is affecting most of the people and it is very difficult to diagnose and treat the diseased patient. Generally asymptomatic patients recover without any treatment. Patients with other illness such as Hypertension, Heart and Lung problems, Diabetic patients require intense care and treatment. In such cases, a team of doctors work together. The combination of all the experts' opinions is needed for efficient treatment. Often, the opinion of doctors depends on their experience and involves some differences. Further, the expert's opinion is in linguistic terms. Plithogenic sets provide a mathematical tool for aggregation of the experts' opinion expressed in linguistic terms. Thus, this work aims to employ plithogenic neutrosophic number to rank the diseased patients affected with COVID-19. Hence, we propose an Order Preference Technique by Similarity to Ideal Solutions (TOPSIS) using Plithogenic sets.

**Keywords:** Plithogenic sets, COVID-19, Medical decision making, TOPSIS method.

### **1. Introduction**

Multi-Criteria Decision Making (MCDM) is applied to numerous pragmatic problems. Approaches to building a dynamic model are additionally differing what's more, rich. The dynamic relies upon the data gathered and the subjectivity of the choice creator. Data might be unclear, wrong and unsure. To this type of situation, neutrosophic  
DOI: 10.5281/zenodo.4011772

Received: May 14, 2020 Accepted: August 30, 2020

number was introduced and the scale neutrosophic sets, was proposed by Smarandache in 1998 [1,2,5], as an integral part to manage incomplete, uncertain and inconsistent data which exist in reality as they are characterized by truth value (T), indeterminacy value (I) and false value (F). This is significant in numerous application zones since indeterminacy is measured expressly and reality participation work, indeterminacy enrollment capacity and misrepresentation participation capacities are free. Wang.et.al in 2010[3,5] introduced the idea of single valued neutrosophic set. The single valued neutrosophic set can autonomously communicate truth-enrollment degree, indeterminacy-participation degree and deception enrollment degree and manages inadequate, vague and conflicting data. All the variables portrayed by the single-valued neutrosophic set are entirely reasonable for human intuition because of the defect of information that human gets or sees from the outside world. Single valued neutrosophic set has been growing quickly because of its wide scope of hypothetical polish and application regions.

Single valued neutrosophic number is an augmentation of fuzzy numbers and intuitionistic fuzzy numbers. Single valued fuzzy number is an extraordinary instance of single valued neutrosophic set and is of significance for dynamic issues.

Application of multi-valued neutrosophic sets in tending to issue with uncertain, imprecise, incomplete and inconsistent data existing in genuine logical and building applications. Tian,et al. in 2016[4,5], characterized the idea of rearranged neutrosophic linguistic sets. Rearranged neutrosophic linguistic sets have empowered incredible advancement in portraying linguistic information to a certain extent.

MCDM is the vital tool for solving problems in real time Decision making (DM). DM is to choose, organize, and rank the limited number of strategies. Since an excessive number of strategies is included, Hwang and Yoon gave a scientific categorization of ordering the procedures by such as: the sorts of data from DMs, striking highlights of data, and a significant class of techniques. This categorisation gives better understanding of MCDM procedures. Among these methods, the categorisation of data based on criteria from DMs with cardinal data is convenient for making decision. In TOPSIS, the idea of separation measures, of the options from the PIS and the NIS was proposed by Hwang and Yoon [6].

Chen.et.al further developed TOPSIS to solve the decision-making problems with different criteria given in fuzzy theory [7]. Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for reducing n-dimensional objective problem into 2-dimensional objective problems in [8]. Ta-Chung.et al. developed a fuzzy TOPSIS model under group decision making to solve the problem of location selection [9]. Shih.et.al, proposed a new method to overcome the problem due to the inappropriately assigned criteria or their weights in MCDM [10]. Shih. et. al, produced a new methodology in normalization operations, distance and mean operators. Moreover, in decision making two or more preferences are aggregated internally in the TOPSIS procedure [11]. Jahanshahloo.et.al, introduced a new TOPSIS for dealing interval data [12]. Liu, P. recommended a TOPSIS method to solve multi-attribute DM problems which depends on its attribute weight [13]. Jadidi.et.al, came out with another strategy dependent on TOPSIS ideas in grey theory to manage the issue of choosing providers [14]. Kao, C. introduced a proportion of relative separation, which includes the figuring of the overall situation of an option between the anti-ideal and the ideal for positioning [15]. Tsaur, R.C. introduced another TOPSIS technique for positioning the choices from the data normalization [16]. Zhang.et.al, built up an improved model to discover the characteristic weights for MADM issues with lacking weight information measures under IVIFSs condition in [17]. Umran.et.al, created the MCDM technique for ranking renewable energy supply systems in Turkey[18]. Sorin.et.al, presented a general view of the developments of fuzzy TOPSIS methods in [19]. Claudia.et.al, introduced ranking strategy for instructional videos by considering choice standards of various characteristics such as exact and loose, and a reference arrangement in [20]. Husin.et.al., introduced ranking the risk variables in [21].

P.Biswas.et.al, introduced decision making in neutrosophic environment [22]. Sorin Nadaban.et.al, A survey on MCDM problems with neutrosophic sets is in [23]. Azeddine.et.al, presented an improved TOPSIS strategy and

extended to simplified neutrosophic - TOPSIS using single valued neutrosophic values [24]. Hagar et al., The proposed procedure opens the entryway of using neutrosophic sets in conjunction with game theory principles in solving competitive MCGDM issues under uncertainty conditions [25]. K. Mondal et al., Introduced, another methodology for MCGDM problems is developed by expanding the TOPSIS technique under rough neutrosophic condition [26]. Akram et al., Presented bipolar neutrosophic TOPSIS technique and bipolar neutrosophic ELECTRE-I strategy in [27]. Xu et al., Introduced, another neutrosophic approach based on TOPSIS technique, which can utilize NS data, is proposed to isolate the designs. Initially, the picture is changed into the NS space. By then, two exercises, modified mean and enhancement tasks are used to overhaul picture edges and to diminish uncertainty [28]. P. Biswas et al., developed nonlinear programming approach in TOPSIS method [29] and weights of DM are dictated by using closeness measure dependent on Hamming distance in [30]. Saqlain et al., explored MCDM problem with multiple-valued neutrosophic data [31]. Azeddine et al. proposed lite TOPSIS from simplified TOPSIS (S-TOPSIS) [32]. A propelled kind of neutrosophic procedure, named type 2 neutrosophic numbers, and characterizes a portion of its operational guidelines in [33]. Nada et al. introduced neutrosophic AHP and TOPSIS to improve the traditional methods of personal selection to achieve the ideal solutions in [34].

Similar to generalisation of fuzzy sets, intuitionistic sets, neutrosophic sets, Florentin Smarandache introduced the new notion of plithogenic sets in [35]. Plithogenic sets whose elements are characterised by multiple attributes is explained in [36]. Extension of plithogenic hypersoft set hyperset is discussed by Florentin Smarandache [37]. Shazia Rana et al. developed matrix representation and operators for plithogenic fuzzy set and plithogenic fuzzy whole hypersoft set [38]. Mohamed Abdel-Basset et al. discussed supply chain problem using plithogenic sets in [39] and also proposed hybrid plithogenic decision making approach [40], a TOPSIS-CITRIC model for supply chain is developed in [41]. Application of plithogenic sets in hospital medical care systems is in [42]. Prem Kumar Singh proposed multivariable data analysis using plithogenic sets in [43].

In this paper, we consider single-valued neutrosophic sets and plithogenic sets. Let  $R$  be a universal set. A single valued neutrosophic set  $D$  on  $R$  is defined as  $D = \{ \langle \delta_D(x), \eta_D(x), \mu_D(x) \rangle : x \in R \}$ . Where  $\delta_D(x), \eta_D(x), \mu_D(x) : R \rightarrow [0, 1]$  represents the membership value degree, indeterministic value degree and non-membership value degree respectively of the elements  $x \in R$  such that  $0 \leq \delta_D(x) + \eta_D(x) + \mu_D(x) \leq 3$ . Every attribute value  $v$  has corresponding (neutrosophic) degree of appurtenance  $d(x, v)$  of the element  $x$  to the plithogenic set  $P$ , with regard to predefined criteria. Further, it includes contradiction degree function to each attribute value with respect to the dominant one. For neutrosophic set, the appurtenance degree  $d : P \times V \rightarrow P([0, 1]^3)$ , contradiction degree  $c : V \times V \rightarrow P([0, 1]^3)$ , for set  $V$  of values of attributes. The proposed method of TOPSIS with plithogenic sets is presented in Section 2. This method is applied to analyse patients with Covid-19 infection, in Section 3 and finally concluded.

## 2. Proposed TOPSIS method for Plithogenic sets

The procedure called TOPSIS (Technique for Order Preference by Similarity to Ideal Situation) can be utilized to assess various choices against the chosen standards. In the TOPSIS approach, an alternative that is closest to the single valued neutrosophic positive ideal solution (SVNPIS) and farthest from the single valued neutrosophic negative ideal solution (SVNNIS) is picked as optimal. An SVNPIS is made out of the best execution esteems for every other option while the SVNNIS comprises of the most noticeably terrible presentation esteems. A point by point depiction and treatment of TOPSIS is examined by [44, 45] and we have adjusted the pertinent strides of TOPSIS using plithogenic sets as introduced beneath. Aggregation of decision makers alternatives and criterion is combined, and the optimal opinion is captured using plithogenic set operations.

Steps for TOPSIS using plithogenic sets:

1. Let there be  $n$ -Decision Makers ( $DM_1, DM_2, DM_3, \dots, DM_n$ ).
2. Each Decision Maker has ' $r$ ' alternatives and ' $s$ ' criterion. The  $l$ -th alternative and  $m$ -th component are  $z_{lm}^n = (\alpha_{lm}^n, \beta_{lm}^n, \gamma_{lm}^n)$  and  $\omega_m^n = (\alpha_m^n, \beta_m^n, \gamma_m^n)$  respectively. where  $l = 1, 2, \dots, r$  and  $m = 1, 2, \dots, s$ .
3. The Plithogenic neutrosophic ratings are aggregated and given as  $z_{lm} = (\alpha_{lm}, \beta_{lm}, \gamma_{lm})$  such that  $(\alpha_{l1}, \alpha_{l2}, \alpha_{l3}) \wedge_p (\beta_{l1}, \beta_{l2}, \beta_{l3}) = \left( \left( \alpha_{l1} \wedge_p \beta_{l1}, \frac{1}{2}(\alpha_{l2} \vee_F \beta_{l2}) + \frac{1}{2}(\alpha_{l2} \wedge_F \beta_{l2}), \alpha_{l3} \vee_p \beta_{l3} \right), 1 \leq l \leq r \right)$  are used for the aggregation of DM's opinion with respect to each criteria.
4. The aggregated Neutrosophic weights of each criterion are calculated as  $\omega_m = (\alpha_m', \beta_m', \gamma_m')$  such that  $\alpha_m' = \min_n \{\alpha_m^n\}, \beta_m' = \frac{1}{N} \sum_{n=1}^s \beta_m^n, \gamma_m' = \max_n \{\gamma_m^n\}$
5. The MCDM problem in matrix format is

$$A = \begin{matrix} & y_1 & y_2 & \cdot & \cdot & \cdot & y_s \\ \begin{matrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_r \end{matrix} & \begin{pmatrix} z_{11} & z_{12} & \cdot & \cdot & \cdot & z_{1s} \\ z_{21} & z_{22} & \cdot & \cdot & \cdot & z_{2s} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ z_{r1} & z_{r2} & \cdot & \cdot & \cdot & z_{rs} \end{pmatrix} \end{matrix}$$

$\omega = (\omega_1, \omega_2, \dots, \omega_n)$  where for all  $z_{lm}$  and  $\omega_m$ ;  $l = 1, 2, \dots, r$  and  $m = 1, 2, \dots, s$ .

Here  $z_{lm} = (\alpha_{lm}, \beta_{lm}, \gamma_{lm})$  and  $\omega_m = (\alpha_m', \beta_m', \gamma_m')$  are neutrosophic numbers representing linguistic variables.

6. Thus we have the normalized Neutrosophic decision matrix as  $M = [d_{lm}]_{r \times s}, l = 1, 2, \dots, r; m = 1, 2, \dots, s$ .

Where  $d_{lm} = \left( \frac{\alpha_{lm}}{\gamma_m^+}, \frac{\beta_{lm}}{\gamma_m^+}, \frac{\gamma_{lm}}{\gamma_m^+} \right)$  and  $\gamma_m^+ = \max_l \gamma_{lm}$  (Benefit criteria)

where  $d_{lm} = \left( \frac{\alpha_m^-}{\gamma_{lm}}, \frac{\alpha_m^-}{\beta_{lm}}, \frac{\alpha_m^-}{\alpha_{lm}} \right)$  and  $\alpha_m^- = \min_l \alpha_{lm}$  (Cost criteria)

The above normalization method preserves the property that the ranges of normalized neutrosophic numbers belongs to  $[0, 1]$ . Either benefit criteria or cost criteria is considered depending on the case study.

7. The weighted normalized neutrosophic decision matrix  $N$  is computed by multiplying the weights  $\omega_m$  of evaluation model with the normalized neutrosophic decision matrix as  $N = [v_{lm}]_{r \times s}$  where

$$v_{lm} = d_{lm}(\cdot) \omega_m = (\alpha_m'', \beta_m'', \gamma_m''), l = 1, 2, \dots, r; m = 1, 2, \dots, s$$

8. The SVNPIs and SVNNIS of the electives are defined as follows

$$P^+ = \{v_1^+, v_2^+, \dots, v_n^+\} \text{ where } v_m^+ = (\gamma, \gamma, \gamma) \text{ such that } \gamma = \max_l \{\gamma_{lm}''\}, l = 1, 2, \dots, r; m = 1, 2, \dots, s.$$

$P^- = \{v_1^-, v_2^-, \dots, v_n^-\}$  where  $v_m^- = (\alpha, \alpha, \alpha)$  such that  $\alpha = \min_l \{\alpha_{lm}^-\}$ ,  $l = 1, 2, \dots, r$ ;  $m = 1, 2, \dots, s$ .

9. The distance  $P_l^+$  and  $P_l^-$  of each weighted alternative  $l = 1, 2, \dots, r$  from the SVNPIs and SVNNIS is computed as follows  $P_l^+ = \sum_{m=1}^r P_v(v_{lm}, v_m^+)$  and  $P_l^- = \sum_{m=1}^r P_v(v_{lm}, v_m^-)$  where  $P_v(i, j)$  is the distance between two single valued Plithogenic neutrosophic numbers 'i' and 'j' i.e., if  $i = (a_1, b_1, c_1)$ ,  $j = (a_2, b_2, c_2)$  then

$$P_v(i, j) = \sqrt{\frac{1}{3} \left\{ (a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 \right\}}.$$

10. The closeness coefficient of  $CC_l$  represents the distance of SVNPIs  $P^+$  and SVNNIS  $P^-$  simultaneously. The closeness coefficients of each alternative is calculated as  $CC_l = \frac{P_l^-}{P_l^+ + P_l^-}$ ,  $l = 1, 2, \dots, r$ .

The proposed TOPSIS method for plithogenic sets is demonstrated to patients suffering from Covid-19.

### 3. Numerical illustration for Covid-19

The whole world is facing and trying to cope up using different strategies to handle the novel CORONA virus (Covid-19). It is well known that while people of all age groups are susceptible to the disease, those with co-morbidities are especially vulnerable to it. For applying the proposed work, Covid-19 affected patients with hypertension, diabetic and heart disease. It is very difficult for a physician to diagnose and appropriately treat such patients. To overcome this, plithogenic neutrosophic linguistic scales are defined based on the diseases and the weights are defined based on the decision maker (Doctors). Let the co-morbidities (hypertension, diabetic and heart disease) be the criteria  $C_1, C_2, C_3$ . Let us take three doctors' opinion and the doctors be the decision-makers  $(DM_1, DM_2, DM_3)$  who will give the opinion or suggestion for hypertension, diabetic and heart disease patient which was measured in neutrosophic scale. Patients with these will also have some other complications, so every patient contradicts with other patients even though, they have a similar type of symptom. Plithogenic concepts are used and the contradiction is recorded. Let the patients be Patient.1, Patient.2, Patient.3, and Patient.4. In Table.1 and Table.2, linguistic variables for describing the intensity of Covid-19 infected patients is presented based on plithogenic number.

**Table.1 Linguistic Variable are defined based on the Disease**

S.No.	Rating Linguistic variable	Plithogenic Number
1	Nothing(N)	(0.11,0.31,0.36)
2	Very Low(VL)	(0.16,0.26,0.11)
3	Low(L)	(0.41,0.36,0.51)
4	Medium(M)	(0.66,0.61,0.71)
5	High(H)	(0.71,0.66,0.81)
6	Very High(VH)	(0.91,0.86,0.91)
7	Absolute(A)	(0.96,0.91,0.96)

**Table.2 Weights of the criteria are defined by the Decision Maker**

S.No.	Linguistic Variables for the Importance Weight of Each Criteria	Plithogenic Number
1	Very Low(VL)	(0.09,0.29,0.34)
2	Low(V)	(0.14,0.24,0.09)
3	Medium Low(ML)	(0.39,0.34,0.49)
4	Medium(M)	(0.64,0.59,0.69)
5	Medium High(MH)	(0.69,0.64,0.79)
6	High(H)	(0.89,0.84,0.89)
7	Very High(VH)	(0.94,0.89,0.94)

The decision makers (doctors) opinion for different patients with weights for each attribute is given in table.3.

**Table.3 Linguistic Variables with Weights**

	Weight	VH	M	VL	H	ML	VH	MH	V	H
Patients (Alternatives)	Contradiction Degree	DM1			DM2			DM3		
		C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
Patient-1	0	N	H	A	VL	A	N	L	VH	H
Patient-2	0.25	VL	M	L	L	H	M	M	VH	L
Patient-3	0.50	VH	A	H	M	VL	L	N	N	A
Patient-4	0.75	L	VH	VL	M	A	VH	N	H	VL

Using step.3 the plithogenic aggregation is calculated with contradiction degree is shown in table.4. For example, the DM's opinion are aggregated in similar form

$$\begin{aligned}
 DM_1 \wedge_p DM_2 &= (0.11, 0.31, 0.36) \wedge_p (0.16, 0.26, 0.11) \\
 &= \left( 0.11 \wedge_p 0.16, \frac{1}{2} (0.31 \vee_F 0.26) + \frac{1}{2} (0.31 \wedge_F 0.26), 0.36 \vee_p 0.11 \right) \\
 &= \left( (1-0) \times (0.11 \times 0.16) + 0, \frac{1}{2} (0.31 + 0.26), (1-0) \times (0.36 + 0.11 - 0.36 \times 0.11) + 0 \right) \\
 &= (0.02, 0.29, 0.43) \\
 DM_1 \wedge_p DM_2 \wedge_p DM_3 &= (0.02, 0.29, 0.43) \wedge_p (0.41, 0.36, 0.51) = (0.01, 0.32, 0.72)
 \end{aligned}$$

**Table.4 Plithogenic aggregation results**

Alternative	Contradiction Degree	DM1^DM2^DM3		
		C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
Patient-1	0	(0.01,0.32,0.72)	(0.62,0.82,1.00)	(0.07,0.64,1.00)
Patient-2	0.25	(0.27,0.46,0.71)	(0.63,0.75,0.93)	(0.29,0.42,0.75)

Patient-3	0.50	(0.45,0.52,0.59)	(0.34,0.45,0.45)	(0.76,0.71,0.81)
Patient-4	0.75	(0.55,0.40,0.30)	(0.91,0.77,0.79)	(0.61,0.41,0.12)

Let the weights of the decision maker be their experience. Their experience is aggregated using step.4 and the weights are calculated as shown in Table-5. i.e. if  $\omega_i = (\alpha'_i, \beta'_i, \gamma'_i)$  then  $\alpha'_1 = \min\{0.94, 0.89, 0.69\} = 0.69$ ,

$$\beta'_1 = \frac{1}{3}(0.89 + 0.84 + 0.64) = 0.79, \gamma'_1 = \max\{0.94, 0.89, 0.79\} = 0.94.$$

Table.5 Weighted decision matrix

Weights	(0.69,0.79,0.94)	(0.14,0.39,0.69)	(0.09,0.67,0.94)
Aggregate decision matrix	DM1^DM2^DM3		
Alternatives	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
Patient-1	(0.01,0.32,0.72)	(0.62,0.82,1.00)	(0.07,0.64,1.00)
Patient-2	(0.27,0.46,0.71)	(0.63,0.75,0.93)	(0.29,0.42,0.75)
Patient-3	(0.45,0.52,0.59)	(0.34,0.45,0.45)	(0.76,0.71,0.81)
Patient-4	(0.55,0.40,0.30)	(0.91,0.77,0.79)	(0.61,0.41,0.12)

In the situation, it is advisable to consider benefit criteria. Thus, the normalized neutrosophic decision matrix is calculated using step.6 as shown in Table-6. In similar form

$$d_{i1} = \left( \frac{\alpha_{i1}}{\gamma_1^+}, \frac{\beta_{i1}}{\gamma_1^+}, \frac{\gamma_{i1}}{\gamma_1^+} \right)$$

where  $\gamma_1^+ = \max\{0.72, 0.71, 0.59, 0.30\} = 0.72$

$$d_{11} = \left( \frac{\alpha_{11}}{\gamma_1^+}, \frac{\beta_{11}}{\gamma_1^+}, \frac{\gamma_{11}}{\gamma_1^+} \right) = \left( \frac{0.01}{0.72}, \frac{0.32}{0.72}, \frac{0.72}{0.72} \right) = (0.01, 0.44, 1.00)$$

Table.6 Normalized Decision Matrix

Weights	(0.69,0.79,0.94)	(0.14,0.39,0.69)	(0.09,0.67,0.94)
	DM1^DM2^DM3		
Alternatives	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
Patient-1	(0.01,0.44,1.00)	(0.62,0.82,1.00)	(1.00,0.64,0.07)
Patient-2	(0.38,0.64,0.99)	(0.63,0.75,0.93)	(0.75,0.42,0.29)
Patient-3	(0.63,0.72,0.82)	(0.34,0.45,0.45)	(0.81,0.71,0.76)
Patient-4	(0.76,0.56,0.42)	(0.91,0.77,0.79)	(0.12,0.41,0.61)

The weighted normalized neutrosophic decision matrix is calculated using step.7 as shown in the Table-7. In similar form

$$v_{11} = d_{11}(\cdot) \omega_1 = (0.01, 0.44, 1.00) \cdot (0.69, 0.79, 0.94) = (0.01, 0.35, 0.94)$$

Table.7 Weighted Normalized decision matrix

Alternatives	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
Patient-1	(0.01,0.35,0.94)	(0.09,0.32,0.69)	(0.09,0.43,0.07)
Patient-2	(0.26,0.50,0.93)	(0.09,0.29,0.64)	(0.07,0.28,0.27)

Patient-3	(0.43,0.57,0.77)	(0.05,0.18,0.31)	(0.07,0.48,0.71)
Patient-4	(0.53,0.44,0.39)	(0.13,0.30,0.55)	(0.01,0.27,0.57)

The SVNPIs is  $\{(0.53, 0.57, 0.94), (0.13, 0.32, 0.69), (0.09, 0.48, 0.71)\}$  and SVNNIS is  $\{(0.01, 0.35, 0.39), (0.05, 0.18, 0.31), (0.01, 0.27, 0.07)\}$  are calculated using step.8. The distance between SVNPIs and SVNNIS is measured using step.9 and closeness coefficients are calculated using step.9 and the patients are ranked as shown in the Table.8.

**Table.8 Distance Measure of Ideal solution and Closeness Coefficients**

Alternatives	Distance of SVNPIs			$P_l^+$	Distance of SVNNIS			$P_l^-$	Closeness Coefficient	Rank
	$C_1$	$C_2$	$C_3$		$C_1$	$C_2$	$C_3$			
Patient-1	0.32	0.02	0.38	0.72	0.32	0.24	0.10	0.65	0.47	4
Patient-2	0.16	0.04	0.28	0.48	0.35	0.20	0.12	0.68	0.59	2
Patient-3	0.11	0.24	0.01	0.36	0.35	0.00	0.39	0.74	0.67	1
Patient-4	0.33	0.08	0.15	0.56	0.30	0.16	0.29	0.76	0.58	3

From Table-8, is the most diseased is patient-3 and is severely affected by Covid-19, which indicates the requirement of critical care and treatment, while Patient -3 is less affected when compared with the others. Thus, TOPSIS method for plithogenic sets can be used to identify the severity of Covid-19 patients.

#### 4. Conclusion

In this paper, we considered the multi-standards choice making, an issue when there is a gathering of decision makers. While crisp data is insufficient to show the real circumstances in MCDM, we changed accessible systems in the TOPSIS strategy when decision-makers used linguistic variables. With respect to estimation of reality, plithogenic sets provide a mean for aggregation of multiple decision makers' opinion. The concept of plithogenic sets is extended to TOPSIS method and demonstrated to the framework of Covid-19.

#### 5. Reference

- [1] Smarandache.F., "A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic", Rehoboth:American Research Press,1998.
- [2] Smarandache. F., "A unifying field in logics: neutrosophic logics", Multiple Valued Logic, 8(3), pp.385-438, 2002.
- [3] Wang, H., Smarandache F., Zhang, Y.Q., Sunderraman. R, "Single valued neutrosophic sets", Multispace and Multistructure, 4, pp. 410-413,2010.
- [4] Tian, Z. P., Wang, J., Zhang, H. Y., Chen, X. H., & Wang, J. Q. "Simplified neutrosophic linguistic normalized weighted Bonferroni mean operator and its application to multi-criteria decision-making problems", pp 3339-3360, 2016.
- [5] Said Broumi, AssiaBakali, Mohamed Talea , Florentin Smarandache, Vakkas Uluçay, Mehmet Sahin, Arindam Dey, Mamouni Dhar, Rui-Pu Tan, Ayoub Bahnasse, Surapati Pramanik, "Neutrosophic Sets: An Overview", New Trends in Neutrosophic Theory and Applications., Volume II, pp.413-444,2017.
- [6] Hwang, C. L.,&Yoon, K. "Methods formultiple attribute decision making", Multiple attribute decision making, pp.58–191, Berlin: Springer,1981.

- [7] C.T. Chen, “Extensions of the TOPSIS for group decision-making under fuzzy environment”, *Fuzzy Sets and Systems*, 114 (1), pp.1–9, 2000.
- [8] Y.J. Lai, “TOPSIS for MODM”, *European Journal of Operational Research* 76, pp.486–500, 1994.
- [9] T.C. Chu, “Facility location selection using fuzzy TOPSIS under group decision”, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 10 (6), pp.687–701, 2002.
- [10] H.S. Shih, C.H. Wang, E.S. Lee, “A multiattribute GDSS for aiding problem-solving”, *Mathematical and Computer Modelling*, 39 (11–12), pp.1397–1412, 2004.
- [11] Shih, H.-S., Shyur, H.-J., & Lee, E. S. “An extension of TOPSIS for group decision making. *Mathematical and Computer Modelling*, 45, pp.801–813, 2007.
- [12] Jahanshahloo, G. R., Hosseinzadeh Lotfi, F., & Davoodi, “A. Extension of TOPSIS for decision-making problems with interval data: Interval efficiency”, *Mathematical and Computer Modelling*, 49, pp.1137–1142, 2009.
- [13] Liu, P. “Multi-attribute decision-making method research based on interval vague set and TOPSIS method” *Technological and Economic Development of Economy*, 15(3), pp.453–463, 2009.
- [14] Jadidi, O., Sai Hong, T., Firouzi, F., & Yusuff, R. M. An optimal grey based approach based on TOPSIS concepts for supplier selection problem. *International Journal of Management Science and Engineering Management*, 4, pp.104–117, 2009.
- [15] Kao, C. ‘Weight Determination for Consistently Ranking Alternatives in Multiple Criteria decision Analysis”, *Applied Mathematical Modelling*, 34, pp.1779–1787, 2010.
- [16] Tsaur, R.-C. “Decision Risk Analysis for an Interval TOPSIS method,” *Applied Mathematics and Computation*, 218, pp.4295–4304, 2011.
- [17] Zhang, H. & Yu, L. “MADM Method based on Cross-entropy and Extended TOPSIS with Interval-valued Intuitionistic Fuzzy Sets,” *Knowledge-Based Systems*, 30, 115–120, 2012.
- [18] Umran S, engul , Miraç Eren, Seyedhadi Eslamian Shiraz , Volkan Gezder, Ahmet Bilal S, engül,” *Fuzzy TOPSIS Method for Ranking Renewable energy supply systems in Turkey*”, Elsevier, *Renewable Energy* 75, 2015.
- [19] Sorin Nadaban, Simona Dzitac & Ion Dzitac, “Fuzzy TOPSIS a General View”, Elsevier B.V, *Procedia Computer Science* 91, pp.823–831, 2016.
- [20] Claudia Margarita Acuña Soto, Vicente Liern, Blanca Pérez-Gladish, “Normalization in TOPSIS-based approaches with data of different nature: application to the ranking of mathematical videos”, *Annals of Operations Research Springer*, 2018.
- [21] Saiful Husin , Fachrurrazi Fachrurrazi, Maimun Rizalihadi, and Mubarak Mubarak, “Implementing Fuzzy TOPSIS on Project Risk Variable Ranking”, *Hindawi Advances in Civil Engineering*, 2019.

- [22] Pranab Biswas, SurapatiPramanik, Bibhas C. Giri, TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment, *Neural Comput&Applic*,2016.
- [23] Sorin Nadaban& Simona Dzitac,”Neutrosophic TOPSIS a General View”, 6th International Conference on Computers Communications and Control (ICCCC),2016.
- [24] AzeddineElhassouny, FlorentinSmarandache, “Neutrosophic-simplified-TOPSIS”, IEEE International Conference on Fuzzy Systems, 2016.
- [25] Hagar G. Abu-Faty, Nancy A. El-Hefnawy, Ahmed Kafafy, “Neutrosophic TOPSIS Based Game Theory for Solving MCGDM Problems”, *Australian Journal of Basic and Applied Sciences*, 11(13), pp. 29-38, 2017.
- [26] Kalyan Mondal, SurapatiPramanik and Florentin Smarandache, “Rough Neutrosophic TOPSIS for Multi-Attribute Group Decision Making”, *Neutrosophic Sets and Systems*, Vol. 13, 2017.
- [27] Muhammad Akram, Shumaiza and Florentin Smarandache, “Decision-Making with Bipolar Neutrosophic TOPSIS and Bipolar Neutrosophic ELECTRE-I”, *Axioms*, 2018, 7, 33,2018.
- [28] G. Xu, S. Wang, T. Yang, W. Jiang, “A Neutrosophic Approach Based on TOPSIS Methodto Image Segmentation”, *International Journal of Computer Communications & Control*, 13(6), pp.1047-1061,2018.
- [29] Pranab Biswas, Surapati Pramanik and Bibhas C. Giri,”NonLinear Programming Approach for Single-Valued Neutrosophic TOPSIS Method”, *New Mathematics and Natural Computation* No. 2, pp.307–326,Vol.15, 2019.
- [30] Pranab Biswas, SurapatiPramanik and Bibhas C. Giri, Neutrosophic TOPSIS with Group Decision Making, *Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets*”, *Studies in Fuzziness and Soft Computing* 369,2019.
- [31] Muhammad Saqlain, Muhammad Saeed, Muhammad Rayees Ahmad, Florentin Smarandache, “Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application", *Neutrosophic Sets and Systems*, Vol. 27, 2019.
- [32] Azeddine Elhassouny, Florentin Smarandache, “Neutrosophic Modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS)”,*Neutrosophic Sets and Systems*, Vol. 24,PP.1-14, 2019.
- [33] Mohamed Abdel-Basset, M. Saleh, Abdualлах Gamal, Florentin Smarandache, “An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number,” *Applied Soft Computing Journal* 77, pp.438–452, 2019.
- [34] Nada A. Nabeeth,Florentin Smarandache, Mohamed Abdel-Basset, Haitham A. El-Ghareeb and Ahamed Aboelfetouh, “An Integrated Neutrosophic-TOPSIS Approach and Its Application to Personnel Selection A New Trend in Brain Processingand Analysis”s, *IEEE Access*, Vol. 7,2019.

- [35] Florentin Smarandache, Plithogeny, Plithogenic Set, Logic, Probability, and Statistics, 2017.
- [36] Florentin Smarandache, “Plithogenic Set, an Extension of Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Sets – Revisited, Neutrosophic Sets and Systems, Vol. 21, pp.153-166, 2018.
- [37] Florentin Smarandache , “Extension of Soft Set to Hypersoft Set, and then to Plithogenic Hypersoft Set,” Neutrosophic Sets and Systems, Vol. 22, pp.168-170, 2018.
- [38] Shazia Rana , Madiha Qayyum, Muhammad Saeed, Florentin Smarandache ,and Bakhtawar Ali Khan,” Plithogenic Fuzzy Whole Hypersoft Set, Construction of Operators and their Application in Frequency Matrix Multi Attribute Decision Making Technique”, Neutrosophic Sets and Systems, Vol. 28, 2019,
- [39] Mohamed Abdel-Basset, Rehab Mohamed, Abd El-Nasser H. Zaied,Abduallah Gamal, Florentin Smarandache, “Solving the supply chain problem using the best-worst method based on a novel Plithogenic model”, Optimization Theory Based on Neutrosophic and Plithogenic Sets, Elseiver,2020.
- [40] Mohamed Abdel-Basset, Rehab Mohamed, Abd El-Nasser H. Zaied and Florentin Smarandache, “A Hybrid Plithogenic Decision-Making Approach with Quality Function Deployment for Selecting Supply Chain Sustainability Metrics”, Symmetry, 2019.
- [41] Mohamed Abdel-Basset, Rehab Mohamed, “A Novel Plithogenic TOPSIS- CRITIC Model for Sustainable Supply Chain Risk Management”, Journal of Cleaner Production”, Vol 247, Elsevier,2020.
- [42] Mohamed Abdel-Basset, Mohamed El-hoseny, Abduallah Gamal, FlorentinSmarandache, “A Novel Model for Evaluation Hospital Medical Care Systems Based on Plithogenic Sets”, Artificial Intelligence In Medicine, 100, 2019.
- [43] Prem Kumar Singh, “Plithogenic Set for Multi-Variable Data Analysis”, International Journal of Neutrosophic Science, Vol. 1, No. 2, pp. 81-89, 2020.
- [44] S. Saghafian and S. Hejazi, ”Multi-criteria Group Decision making using a Modified Fuzzy topsis Procedure”,vol. 2, pp. 215 –221, nov. 2005.
- [45] Yan-Ping Jiang, Zhi-Ping Fan, Jian Ma., “A Method for Group Decision Making with Multigranularity Linguistic Assessment Information”, Information Sciences, vol.178, no. 4, pp. 1098–1109, 2008.